Proceedings of DETC'05
2005 ASME Design Engineering Technical Conferences
Long Beach, California, USA, September 24-28, 2005

DETC2005-84871

SOME SPECIAL CASES OF THE BURMESTER PROBLEM FOR FOUR AND FIVE POSES

Jorge Angeles and Shaoping Bai
Department of Mechanical Engineering &
Centre for Intelligent Machines
McGill University, Canada
E-mail: {angeles, mspbai }@cim.mcgill.ca

ABSTRACT

The Burmester problem aims at finding the geometric parameters of a planar four-bar linkage for a prescribed set of finitely separated poses. The synthesis related to the Burmester problem deals with both revolute-revolute (RR) and prismatic-revolute (PR) dyads. A PR dyad is a special case of RR dyad, i.e., a dyad with one end-point at infinity. The special nature of PR dyads warrants a special treatment, outside of the general methods of four-bar linkage synthesis, which target mainly RR dyads. In this paper, we study the synthesis of planar four-bar linkages addressing the problem of the determination of PR dyads. The conditions for the presence of PR dyads with the prescribed poses are derived. A synthesis method is developed by resorting to the parallelism condition of the displacement vectors of the circle points of PR dyads. We show that the "circle" point of a PR dyad can be determined as one common intersection of three or four circles, depending on whether four or, correspondingly, five poses are prescribed.

Keywords: Burmester problem, four-bar linkage, parallelism condition, prismatic-revolute dyads.

1 Introduction

The Burmester problem aims at finding the geometric parameters of a four-bar linkage for a prescribed set of finitely separated poses[1]. It is well-known that a RR dyad can be synthesized exactly for up to five prescribed poses. The synthesis problem discussed here pertains to both four and five poses.

The four-pose problem can be solved geometrically. Moreover, the four-pose problem is known to admit infinitely many solutions, each solution dyad being given by a pair of corresponding cubics, the *centerpoint* and the *circlepoint* curves. The five-pose problem, on the other hand, has to be solved numerically. Extensive research has been reported on solution of the Burmester problem with different approaches. Bottema and Roth [2], McCarthy [3] and Hunt [4] solved the problem by intersecting two centerpoint curves of two four-pose problems for two subsets of four poses out of the given five-pose set, to obtain the centers. Beyer [5] and Lichtenheldt [6] reported a method based on projective geometry, while Modler [7–10] investigated various special cases. Al-Widyan and Angeles [11] developed a robust algorithm to synthesize four-bar linkages, in which circle points and center points were found through the intersections of the four possible contours of the four-pose problems. Sandor and Erdman applied complex numbers [12], while Ravani and Roth [13], Hayes and Paul [14] solved the problem via the kinematic mapping. Of all works cited, most focus on revolute-revolute (RR) dyads, few investigating the problem of prismatic-revolute (PR) dyads. To the authors' knowledge, the feature of four-bar linkage synthesis with PR

[1] "Are there any points in a rigid body whose corresponding positions lies on a circle of the fixed plane for the four arbitrarily prescribed positions?" [1]

dyads is not included in the available design software, such as LINCAGES [15].

In this paper we study the synthesis of planar four-bar linkages addressing the problem of the determination of PR dyads. The conditions for the presence of PR dyads within the prescribed set of poses are derived. A synthesis method is developed by resorting to the parallelism condition of the displacement vectors of the circle points of a PR dyad. Some examples are provided to validate the proposed method.

2 Problem Formulation

The Burmester problem reads: A rigid body, attached to the coupler link of the four-bar linkage, as shown in Fig. 1a, is to be guided through a discrete set of m poses, given by $\{\mathbf{r}_j,\, \theta_j\}_0^m$, where $\mathbf{r}_j$ is the position vector of a landmark point R of the body at the jth pose and θ_j is the corresponding angle of a line of the body, as depicted in Fig. 2. The problem consists in finding the joint centers A_0 and B that define the BA_0R *dyad* of the guiding four-bar linkage, dyad $B^*A_0^*R$ being determined likewise. Given that A_0 and A_0^* describe circles centered at B and B^*, respectively, the former are termed the *circle points*, the latter the *center points* of the dyads.

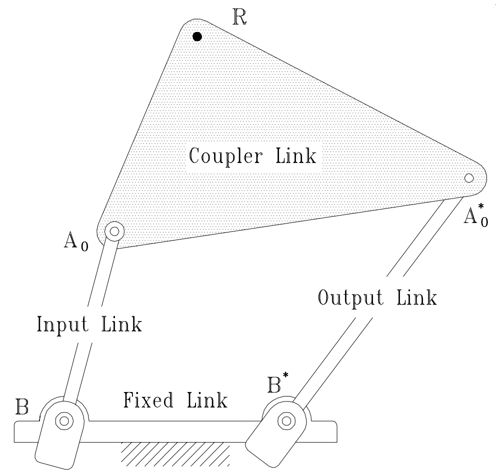
If a PR dyad exists for the prescribed poses, as depicted in Fig. 1b, the center-point B^* is located at infinity. For $m = 3, 4$, a discrete set of circle point solutions exists, besides the direction corresponding to the prismatic joint. For RR dyads, we have:

> In the case of $m = 3$, the problem leads to a system of three algebraic equations in four unknowns, the coordinates of points A_0 and B. Hence, infinitely many solutions are available; these solutions defining two related loci: the *centerpoint* and the *circlepoint* (cubic) curves.
> In the case of $m = 4$, a system of four algebraic equations exists for four unknowns, the coordinates of points A_0 and B. The problem is thus determined, but nonlinear, and admits up to four different solutions, i.e., four different dyads. The pairwise combination of these dyads then leads to up to six distinct linkages.

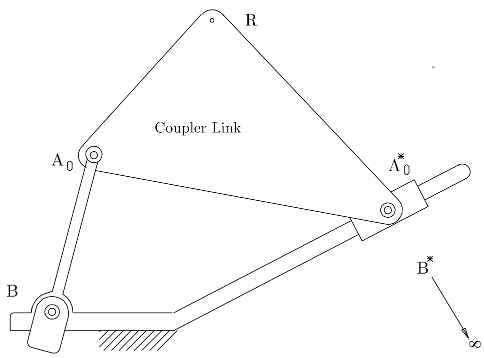
In the balance of this paper, we will develop two general synthesis methods, one for four and one for five poses, applicable to problems admitting one PR dyad.

3 Synthesis with Four Prescribed Poses

A PR dyad is also considered as a RR dyad with one point at infinity. Without loss of generality, we start the synthesis with a general four-bar linkage, as shown in Fig. 2. In the case of four prescribed poses, we have $m = 3$. Under the usual rigid-body assumption, the synthesis equation is readily derived:



(a)



(b)

Figure 1. Four-bar linkages with: (a) Revolute-Revolute dyads only; and (b) one Prismatic-Revolute dyad

$$\| \underbrace{(\mathbf{r}_j - \mathbf{b}) + \mathbf{Q}_j\mathbf{a}_0}_{\mathbf{a}_j-\mathbf{b}} \|^2 = \|\mathbf{a}_0 - \mathbf{b}\|^2\,, \qquad \text{for } j = 1, 2, 3 \tag{1a}$$

where $\mathbf{a}_0$ and $\mathbf{b}$ are position vectors of points A_0 and B, the design parameters of the linkage. $\mathbf{Q}_j$ denotes the rotation matrix carrying the guided body from pose 0 to pose j, i.e.,

$$\mathbf{Q}_j = \begin{bmatrix} \cos\phi_j & -\sin\phi_j \\ \sin\phi_j & \cos\phi_j \end{bmatrix} \tag{1b}$$

with $\phi_j \equiv \theta_j - \theta_0$.

Upon expansion of Eq. (1a) and simplifying the expres-

Copyright © 2005 by ASME

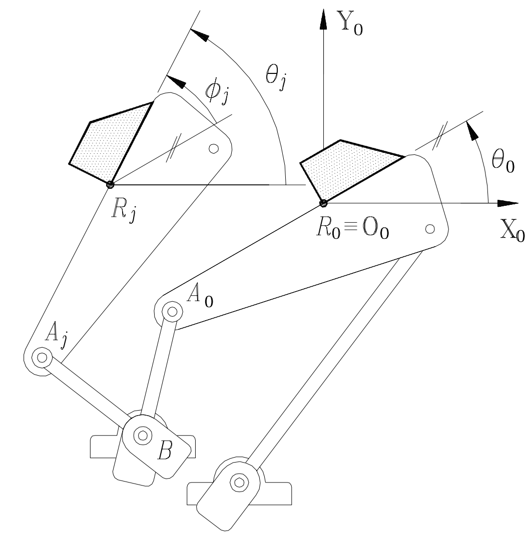


Figure 2. Two finitely-separated poses of a rigid body carried by the coupler link of a four-bar linkage

sion thus resulting, we obtain

$$\mathbf{b}^T(\mathbf{1}-\mathbf{Q}_j)\mathbf{a}_0+\mathbf{r}_j^T\mathbf{Q}_j\mathbf{a}_0-\mathbf{r}_j^T\mathbf{b}+\frac{\mathbf{r}_j^T\mathbf{r}_j}{2}=0\,,\quad j=1,2,3 \tag{2}$$

which are the three *synthesis equations* allowing us to compute the design parameters.

To accommodate both PR and RR dyads, we express **b** in *homogeneous-coordinate* form. To this end, we define

$$\mathbf{b}\equiv\frac{\boldsymbol{\beta}}{w},\qquad w=1\quad\text{or}\quad 0 \tag{3}$$

In Eq. (3), $w=1$ applies to RR dyads, in which a finite point exists for B. When $w=0$, $\mathbf{b}$ corresponds to a point at infinity, which leads to a PR dyad. Under this condition, we define $\|\boldsymbol{\beta}\|=1$, in that a point at infinity has only a "direction," but no position.

Upon substitution of Eq. (3) into Eq. (2), and clearing denominators, we obtain

$$[(\mathbf{1}-\mathbf{Q}_j)\mathbf{a}_0-\mathbf{r}_j]^T\boldsymbol{\beta}+w\left(\mathbf{r}_j^T\mathbf{Q}_j\mathbf{a}_0+\frac{\mathbf{r}_j^T\mathbf{r}_j}{2}\right)=0\,,\qquad j=1,2,3 \tag{4}$$

With $w=0$, Eq. (4) becomes

$$\mathbf{u}_j^T\boldsymbol{\beta}=0,\qquad j=1,\ldots,3 \tag{5}$$

where $\mathbf{u}_j\equiv\mathbf{a}_j-\mathbf{a}_0$ is the displacement of the circle point A_0 at the jth pose, i.e.,

$$\mathbf{u}_j=\mathbf{r}_j-(\mathbf{1}-\mathbf{Q}_j)\mathbf{a}_0 \tag{6}$$

.

3.1 Determination of the Parallelism Condition

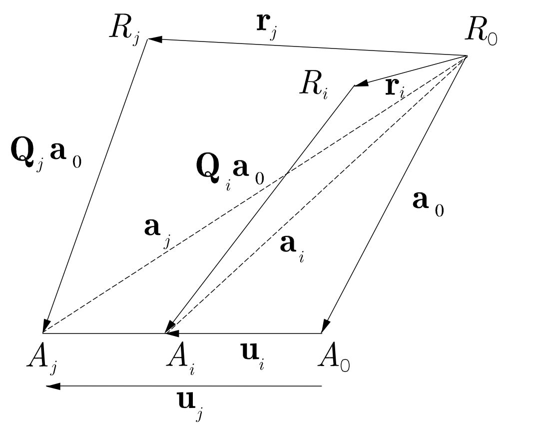


Figure 3. Relation between the pose and the circle points

With reference to Fig. 3, $\mathbf{u}_i$ $(i=1,2,3)$ is a displacement vector of the circle point. If a PR dyad is possible, all three vectors must be parallel. In other words, the cross products of any two displacement vectors must vanish. However, rather than working with cross products, we simplify the analysis by resorting to the two-dimensional representation of the cross product introduced in [16]. This is based on matrix $\mathbf{E}$ rotating vectors in the plane through $90°$ ccw. Hence, the parallelism condition between $\mathbf{u}_i$ and $\mathbf{u}_j$ can be expressed as

$$\Delta_{ij}=\mathbf{u}_i^T\mathbf{E}\mathbf{u}_j=0,\qquad i,j=1,2,3,\qquad i\neq j \tag{7}$$

which expands to

$$\begin{aligned}\Delta_{ij}&=\mathbf{a}_0^T(-\mathbf{E}\mathbf{Q}_i-\mathbf{Q}_j^T\mathbf{E}+\mathbf{Q}_j^T\mathbf{E}\mathbf{Q}_i)\mathbf{a}_0\\&\quad+(\mathbf{E}\mathbf{r}_i-\mathbf{Q}_j^T\mathbf{E}\mathbf{r}_i-\mathbf{E}\mathbf{r}_j+\mathbf{Q}_i^T\mathbf{E}\mathbf{r}_j)^T\mathbf{a}_0+\mathbf{r}_j^T\mathbf{E}\mathbf{r}_i=0\end{aligned} \tag{8}$$

We develop below all terms of the first line of Eq. (8) by writing $\mathbf{Q}_i$ in the form $\mathbf{Q}_i=c_i\mathbf{1}+s_i\mathbf{E}$, in which $\mathbf{1}$ is the 2×2

Copyright © 2005 by ASME

identity matrix, while $s_i \equiv \sin\phi_i$ and $c_i \equiv \cos\phi_i$. Hence,

$$\begin{aligned}
-\mathbf{a}_0^T \mathbf{E}\mathbf{Q}_i \mathbf{a}_0 &= -\mathbf{a}_0^T \mathbf{E}(c_i \mathbf{1} + s_i \mathbf{E})\mathbf{a}_0 \\
&= -c_i \mathbf{a}_0^T \mathbf{E}\mathbf{a}_0 - s_i \mathbf{a}_0^T \mathbf{E}^2 \mathbf{a}_0 \\
&= s_i ||\mathbf{a}_0||^2 \qquad (9a) \\
-\mathbf{a}_0^T \mathbf{Q}_j^T \mathbf{E}\mathbf{a}_0 &= -\mathbf{a}_0^T \mathbf{E}^T \mathbf{Q}_j \mathbf{a}_0 \\
&= \mathbf{a}_0^T \mathbf{E}\mathbf{Q}_j \mathbf{a}_0 \\
&= -s_j ||\mathbf{a}_0||^2 \qquad (9b) \\
\mathbf{a}_0^T \mathbf{Q}_j^T \mathbf{E}\mathbf{Q}_i \mathbf{a}_0 &= \mathbf{a}_0^T (c_j \mathbf{1} - s_j \mathbf{E})(c_i \mathbf{E} - s_i \mathbf{1})\mathbf{a}_0 \\
&= \mathbf{a}_0^T [-(c_j s_i - s_j c_i)\mathbf{1} + (c_j s_i + s_j c_i)\mathbf{E}]\mathbf{a}_0 \\
&= (-c_j s_i + s_j c_i)||\mathbf{a}_0||^2 \\
&= \sin(\phi_i - \phi_j)||\mathbf{a}_0||^2 \qquad (9c)
\end{aligned}$$

Further, let $\mathbf{v}_{ij} = -\mathbf{E}\mathbf{r}_i + \mathbf{Q}_j^T \mathbf{E}\mathbf{r}_i + \mathbf{E}\mathbf{r}_j - \mathbf{Q}_i^T \mathbf{E}\mathbf{r}_j$, as appearing in the second line of Eq. (8), which is now rewritten as

$$\Delta_{ij} = (s_i - s_j + s_{i\bar{j}}) \parallel \mathbf{a}_0 \parallel^2 + \mathbf{v}_{ij}^T \mathbf{a}_0 + \mathbf{r}_j^T \mathbf{E}\mathbf{r}_i = 0, \quad i, j = 1, 2, 3, \quad i \neq j \qquad (10)$$

where $s_{i\bar{j}} = \sin(\phi_i - \phi_j)$. Notice that Eq. (10) represents a family of three circles, This equation can be cast in linear homogeneneous form as

$$\mathbf{M}\mathbf{x} = \mathbf{0}_3 \qquad (11a)$$

with $\mathbf{0}_3$ denoting the three-dimensional zero vector, and

$$\mathbf{x} = [\,||\mathbf{a}_0 \parallel^2 \quad \mathbf{a}_0^T \quad 1\,]^T \qquad (11b)$$

$$\mathbf{M} = \begin{bmatrix} s_1 - s_2 + s_{1\bar{2}} & \mathbf{v}_{12}^T & \mathbf{r}_2^T \mathbf{E}\mathbf{r}_1 \\ s_2 - s_3 + s_{2\bar{3}} & \mathbf{v}_{23}^T & \mathbf{r}_3^T \mathbf{E}\mathbf{r}_2 \\ s_3 - s_1 + s_{3\bar{1}} & \mathbf{v}_{31}^T & \mathbf{r}_1^T \mathbf{E}\mathbf{r}_3 \end{bmatrix} \qquad (11c)$$

It is noted that $\mathbf{M}$ is a 3×4 matrix. Because of the parallelism condition, only two out of the three equations of Eq. (10) are independent, which implies $rank(\mathbf{M}) < 3$. This leads to the result:

Let $\mathbf{M}_i$ be the 3×3 submatrix of $\mathbf{M}$, formed by deleting the ith column from $\mathbf{M}$. If a PR dyad exists, then the determinant of $\mathbf{M}_i$ vanishes, i.e.,

$$\det(\mathbf{M}_i) = 0, \quad i = 1, \ldots, 4 \qquad (12)$$

The converse of the above statement is true:

If $\det(\mathbf{M}_i) = 0$, for $i = 1, \ldots, 4$, then a PR dyad is possible.

Therefore, we can use Eq. (12) to detect whether a *center point* lies at infinity, i.e., whether a PR dyad is possible.

3.2 Determination of PR Dyads

Once the pose data lead to a centerpoint at infinity, the synthesis is bound to lead to a unique $\mathbf{a}_0$. The value of $\mathbf{a}_0$ is found by solving the three equations of Eq. (10). Notice that the x and y components of $\mathbf{a}_0$ can be obtained by plotting the contours defined by these three equations in the x-y plane, and finding their intersections graphically.

Once $\mathbf{a}_0$ is obtained, solving for $\boldsymbol{\beta}$ is straightforward:

$$\boldsymbol{\beta} = \mathbf{E}\hat{\mathbf{u}} \qquad (13)$$

where $\hat{\mathbf{u}}$ is the unit vector obtained from $\mathbf{u}$, which is defined as the mean of the set $\{\mathbf{u}_j\}_1^m$, i.e.[2],

$$\mathbf{u} = \frac{1}{m}\sum_{j=1}^{m} \mathbf{u}_j \qquad (14)$$

4 Synthesis with Five Prescribed Poses

With five given poses, the synthesis equations of a four-bar linkage become

$$[(\mathbf{1}-\mathbf{Q}_j)\mathbf{a}_0 - \mathbf{r}_j]^T \boldsymbol{\beta} + w\left(\mathbf{r}_j^T \mathbf{Q}_j \mathbf{a}_0 + \frac{\mathbf{r}_j^T \mathbf{r}_j}{2}\right) = 0\,, \quad j = 1, \ldots, 4 \qquad (15)$$

Now we have four displacement vectors, as given by Eq. (6), for $j = 1, \ldots, 4$. All four displacement vectors $\mathbf{u}_j$ are parallel if a PR exists, i.e.,

$$\Delta_{ij} = \mathbf{u}_i^T \mathbf{E}\mathbf{u}_j = 0, \quad i, j = 1, \ldots, 4, \quad i \neq j \qquad (16)$$

Similar to the case of four poses in Section 3, we have now:

$$\Delta_{ij} = s_i - s_j + s_{i\bar{j}}) \parallel \mathbf{a}_0 \parallel^2 + \mathbf{v}_{ij}^T \mathbf{a}_0 + \mathbf{r}_j^T \mathbf{E}\mathbf{r}_i = 0, \quad i, j = 1, \ldots, 4, \quad i \neq j \qquad (17)$$

[2]One single vector $\mathbf{u}_j$ would suffice. We take the mean value in order to filter out round-off error.

Copyright © 2005 by ASME

which is, again, a family of circles, this time comprising four circles. This equation can be further rewritten as

$$\mathbf{N}\mathbf{x} = \mathbf{0}_4 \tag{18a}$$

with

$$\mathbf{x} = [\, \| \mathbf{a}_0 \|^2 \quad \mathbf{a}_0^T \quad 1\,]^T \tag{18b}$$

$$\mathbf{N} = \begin{bmatrix} s_1 - s_2 + s_{1\bar{2}} & \mathbf{v}_{12}^T & \mathbf{r}_2^T \mathbf{E} \mathbf{r}_1 \\ s_2 - s_3 + s_{2\bar{3}} & \mathbf{v}_{23}^T & \mathbf{r}_3^T \mathbf{E} \mathbf{r}_2 \\ s_3 - s_4 + s_{3\bar{4}} & \mathbf{v}_{34}^T & \mathbf{r}_4^T \mathbf{E} \mathbf{r}_3 \\ s_4 - s_1 + s_{4\bar{1}} & \mathbf{v}_{41}^T & \mathbf{r}_1^T \mathbf{E} \mathbf{r}_4 \end{bmatrix} \tag{18c}$$

Due to the parallelism of the four vectors $\{\mathbf{u}_j\}_1^4$, the 4×4 matrix **N** is singular, which means its determinant vanishes, i.e.,

$$\det(\mathbf{N}) = 0 \tag{19}$$

Two specials cases can be identified for which $\mathbf{N}$ is singular:

(a) The first column vanishes. This case occurs when the orientation of the guided body remains constant. A possible solution is the parallelogram mechanism.
(b) The last column vanishes. In this case, all given displacement vectors are parallel, while the circle point A_0 coincidents with R_0 and, hence, with the origin O_0.

Based on the above considerations, the determination criteria for the existence of PR dyads is:

$$\det(\mathbf{N}) = 0, \quad \text{with } \mathbf{c}_1 = 0, \quad \mathbf{c}_4 \neq 0. \tag{20}$$

where $\mathbf{c}_1$ and $\mathbf{c}_4$ are the first and fourth columns of $\mathbf{N}$, respectively.

The synthesized PR dyad can be determined upon solving Eq. (17) for $\mathbf{a}_0$. It is noteworthy that four equations of Eq. (17) can regarded as two sets of bivariate equations. Once the parallelism condition is found to hold, both sets of equations must have a set of common solutions, which is the initial position of the circle point, given by $\mathbf{a}_0$. Finally, the direction vector $\boldsymbol{\beta}$ is obtained from Eq. (13).

5 Numerical Examples

We provide two examples to illustrated the foregoing synthesis procedure. Prior to embarking on the numerical solution of the synthesis equations, we normalize the equations to render them dimensionless, thereby avoiding loss of precision.

5.1 Normalization of the Displacement Vector

One may realize that the numerical results of $\det(\mathbf{M}_i)$ of Eq. (12) or $\det(\mathbf{N})$ of Eq. (19) are not exactly zero due to round-off error, which is bound to be amplified because of the presence of dimensions in the data. To reduce the effect of data-dimension on round-off error, normalized displacement vectors are desirable. To this end, we make these vectors dimensionless by means of normalization of the displacement.

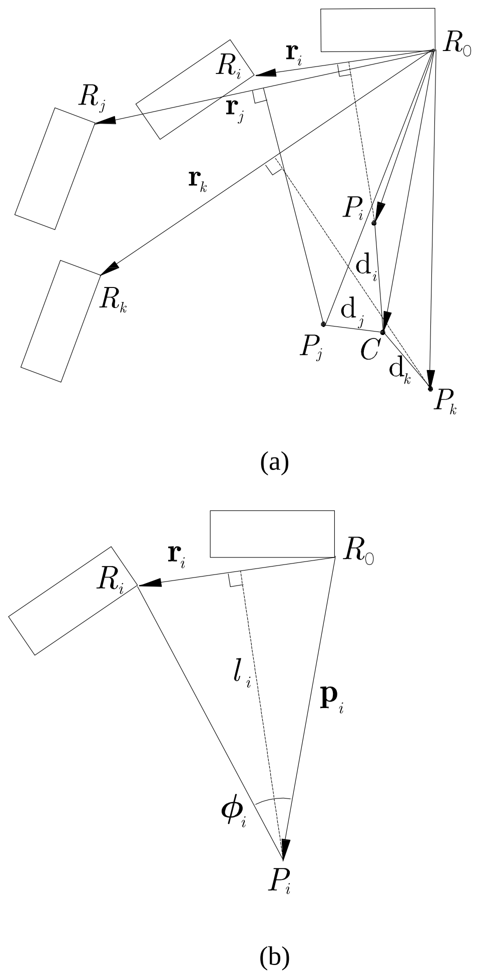


Figure 4. Illustration of vector normalization: (a) determination of the centroid, (b) determination of pole P_i

As depicted in Fig. 4a, the displacement from the reference pose to the jth pose can be considered as a pure rotation about the pole P_j. After taking the centroid C of all P_j, distances $\{d_j\}_1^m$ between P_j and the centroid are obtained. The rms value d_{rms} of these distances is considered as the *characteristic length* for normalization, such that the displacement

Copyright © 2005 by ASME

vectors are normalized as

$$\mathbf{r}'_j \equiv \frac{\mathbf{r}_j}{d_{rms}}, \quad j=1,\ldots,m \tag{21}$$

with

$$d_{rms} = \sqrt{\frac{1}{m}\sum_{j=1}^{m} d_j^2} \tag{22a}$$

$$d_j = \| \mathbf{p}_j - \mathbf{c} \| \tag{22b}$$

$$\mathbf{c} = \frac{1}{m}\sum_{j=1}^{m} \mathbf{p}_j \tag{22c}$$

To find $\mathbf{p}_j$, we refer to Fig. 4b, whence,

$$\mathbf{p}_j = \frac{\mathbf{r}_j}{2} + l_j \mathbf{E}\hat{\mathbf{r}}_j, \quad j=1,\ldots,m \tag{23a}$$

where $\hat{\mathbf{r}}_j$ is the unit vector obtained from $\mathbf{r}_j$ and

$$l_j = \frac{\| \mathbf{r}_j \|}{2\tan(\phi_j/2)} \tag{23b}$$

5.2 Synthesis of PR Dyads

5.2.1 Example 1 The first example is the synthesis of a four-bar linkage guiding its coupler link through the five poses of Table 1. The determinant of matrix of $\mathbf{M}$, calculated with normalized displacement vectors, is equal to 0.8×10^{-18}, a small-enough value to be taken as zero, which means a PR dyads exists. In finding the value of $\mathbf{a}_0$ and $\boldsymbol{\beta}$, four circle equations are obtained from Eq. (17):

$$\Delta_{12} = -0.0046389964x^2 - .71629323x - 0.0046389964y^2 - .29517702y + 5.2135630 \tag{24a}$$

$$\Delta_{23} = 0.00.2389271x^2 - 1.8832828x + 0.002389271y^2 - .97913984y + 11.41245274 \tag{24b}$$

$$\Delta_{34} = 0.0032202972x^2 - 3.2698562x + 0.0032202972y^2 - 1.6663437y + 20.05563909 \tag{24c}$$

$$\Delta_{41} = 0.0015609597x^2 - 3.3158753x + 0.0015609597y^2 - 1.4364879y + 21.652795 \tag{24d}$$

The solution of these four equations is the unique value of $\mathbf{a}_0$, which can be found from inspection of Fig. 5, where all four circles intersect at one common point. With $\mathbf{a}_0$ known, $\boldsymbol{\beta}$ is determined by Eq. (13). The results are recorded in Table 2.

Table 1. Five prescribed poses for rigid-body guidance

j	$\mathbf{r}_j$	ϕ_j [deg]
0	$[0.0, 0.0]^T$	0.0
1	$[-2.70366628, .650818622]^T$	-13.30931971
2	$[-7.30557983, -.169753064]^T$	-19.93765489
3	$[-12.19926854, -1.845618865]^T$	-17.30994130
4	$[-16.23615298, -4.100359719]^T$	-6.529901368

In finding the RR dyad of the four-bar linkage, we resort to the method proposed in [11]. By eliminating $\mathbf{a}_0$ from Eq. (2), four equations in $\mathbf{b}$ are obtained. The solution of the set of equations yields the values of $\mathbf{b}$, as listed in Table 2. Of the three solutions, corresponding to the three intersections in Fig. 6, only one value is in agreement with the prescribed poses. Solution #1 in the list was selected by inspection,. As depicted in Fig. 7, the coupler link of the synthesized mechanism passes through all five prescribed poses.

5.2.2 Example 2 In the second example, the synthesis with four poses is conducted by taking the first four pose data of Table 1. The determinants of the three submatrices are recorded in Table 3. It is seen that all values are very small to suggest the existence of a PR dyad. The solution of the PR dyad is obtained from the common intersection of the three circles of Fig. 8a, which are plotted with Eq. (10). Not surprisingly, the result of the PR dyad in this example is the same as that of the five poses obtained in Example 1. As for the RR dyads, infinitely many solutions for $\mathbf{a}_0$ and $\mathbf{b}$ are available, namely, any point on the centerpoint and circlepoint curves in Fig. 8c.

6 Conclusions

A method to determine PR dyads in the synthesis of four-bar linkages was introduced as special cases of the Burmester problem, for four and five prescribed poses. We derived the criteria for determining the displacement parallelism conditions and developed the procedure to find the unique orientation of the prismatic joint. We proved that the "circle" point of a PR dyad is located at the common intersection of three or four circles, depending on whether four or five poses are prescribed. The method introduced here can be integrated with special-purpose software, e.g., LINCAGES [15], for mechanism design.

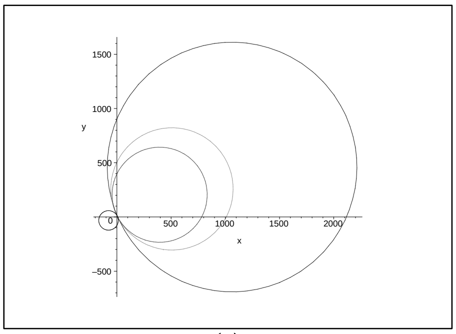
Copyright © 2005 by ASME

Table 2. Synthesis of a four-bar linkage

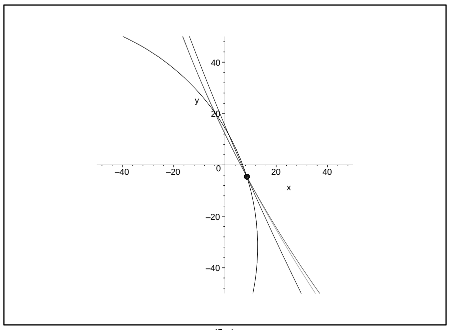
		$\mathbf{a}$	$\boldsymbol{\beta}$	w
PR dyad		[8.549330973, -4.559162957]	[-.2873478855, .9578262853]	0
RR dyad	#1	[-9.166635761, -7.744210073]	[-16.82708020, -14.17208613	1
	#2	[10.51078287, -2.357687934]	[-12.08978824, 39.97541201]	1
	#3	[23.74882468, 8.993497071]	[14.72165227, 8.573517789]	1

Table 3. Values of determinants

j	1	2	3	4
$\delta_j [\times 10^{-6}]$	-0.4182203253	0.481755184	0.256909038	7.12759032



(a)



(b)

Figure 5. Contour plots to determine the circler point of a PR dyad: (a) the big picture, (b) a zoom-in around the common intersection of all contours.

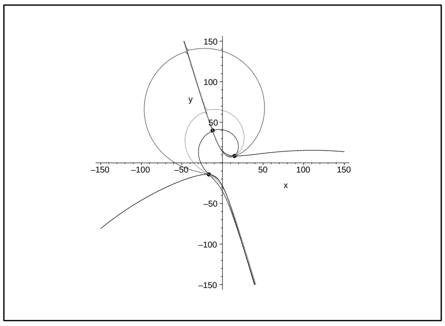


Figure 6. Contour plots to find the center point of a RR dyad

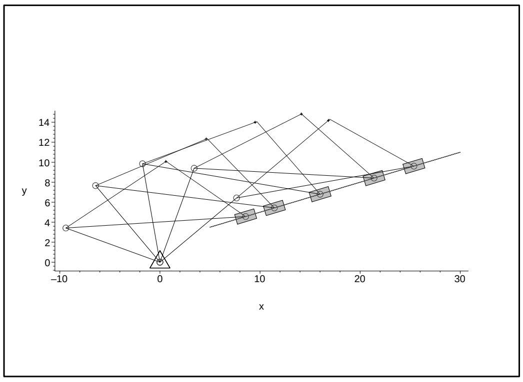


Figure 7. The linkage synthesized for the five prescribed poses

REFERENCES

[1] Burmester, L., 1888. *Lehrbuch der Kinematik*. Arthur Felix Verlag, Leipzig, Germany.

[2] Bottema, O., and Roth, B., 1979. *Theoretical Kinematics*. North-Holland Pub. Co., New York.

[3] McCarthy, J. M., 2000. *Geometric Design of Linkages*. Springer-Verlag, New York.

[4] Hunt, K. H., 1978. *Kinematic Geometry of Mechanisms*. Oxford University Press, New York.

[5] R. Beyer, 1953 *Kinematische Getriebesynthese*. Berlin-Göttingen-Heidelbrg: Spinger Verlag.

[6] W. Lichtenheldt, 1967 *Konstrktionslehre der Getriebe*. Akademie-Verlag, Berlin.

[7] K.-H. Modler, 1972. Beitrag zur theorie der burmesterschen mittelpunktkurve, –Teil 1. *Maschinenbautechnik*,

Copyright © 2005 by ASME

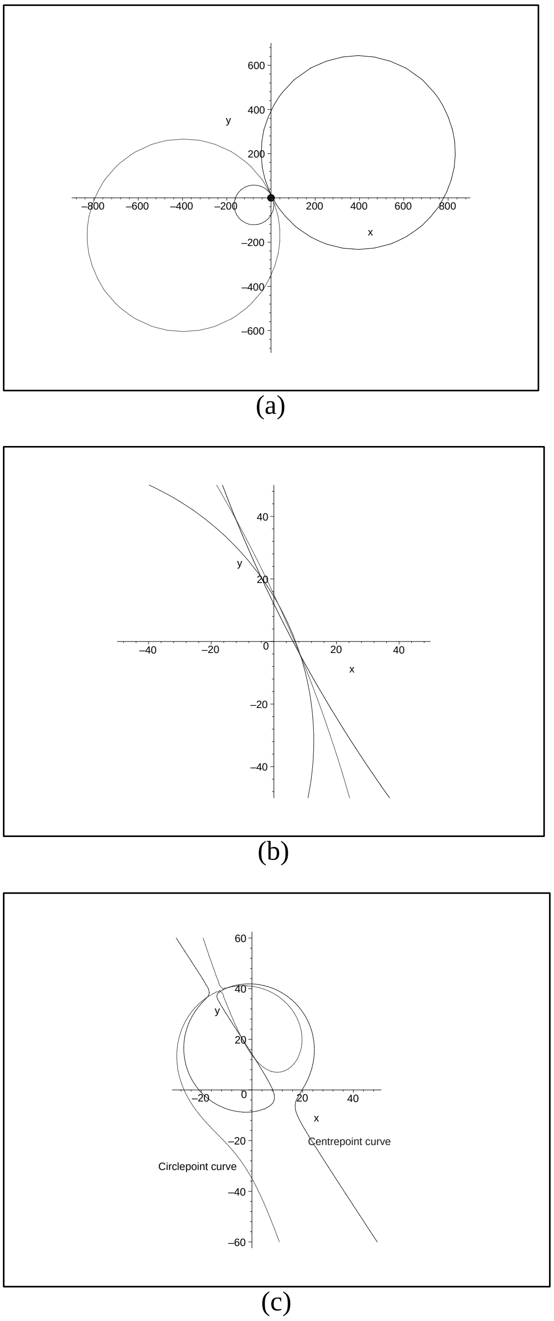


Figure 8. Contour plots for dyad synthesis with four poses: (a) determination of the circle point of the PR dyad; (b) zoomed-in circlepoint contours; (c) contours of center- and circle points of the RR dyad

21(3):98–102

[8] K.-H. Modler, 1972. Erzeugung von sonderfällen der Mittelpunktkurve, –Teil 2. *Maschinenbautechnik*, 21(4):151-154.

[9] K.-H. Modler, 1972. Sonderfällen der Mittelpunktkurve, –Teil 3. *Maschinenbautechnik*, 21(5):199-202.

[10] K.-H. Modler, 1972. Reihenfolge der homologen punkte bei den sonderfällen der Mittelpunktkurve, –Teil 5. *Maschinenbautechnik*, 21(7):293-296.

[11] Al-Widyan, K., Angeles, J., and Cervantes-Sánchez, J. J., 2002. “A numerical roboust algorithm to solve the five-pose burmester problem”. Proc. of DETC’2002, #MECH–34270.

[12] Sandor, G. N., and Erdman, A., 1984. *Advanced Mechanism Design: Analysis and Synthesis*, vol. 2. Prentice-Hall, Inc., New Jersey.

[13] Ravani, B., and Roth, B., 1983. “Motion synthesis using kinematic mappings”. *ASME Journal of Mechanism, Transmissions, and Automation in Design*, **105** , pp. 460–467.

[14] Hayes, M., and Zsombor-Murray, P., 2002. “Solving the burmester problem using kinematic mapping”. Proc. of DETC’2002, #MECH–34378.

[15] Chidambaran, N. C. and Erdman, A. G., 2000. “LINCAGES 2000 Mechanism Design Software”. Proc. Sixth Applied Mechanisms and Robotics Conference.

[16] Sinatra, R., and Angeles, J., 2003. “A novel approach to the teaching of planar mechanism dynamics. a case study”. The International Journal of Mechanical Engineering Education, **31** (3) , pp. 201–214.

Copyright © 2005 by ASME