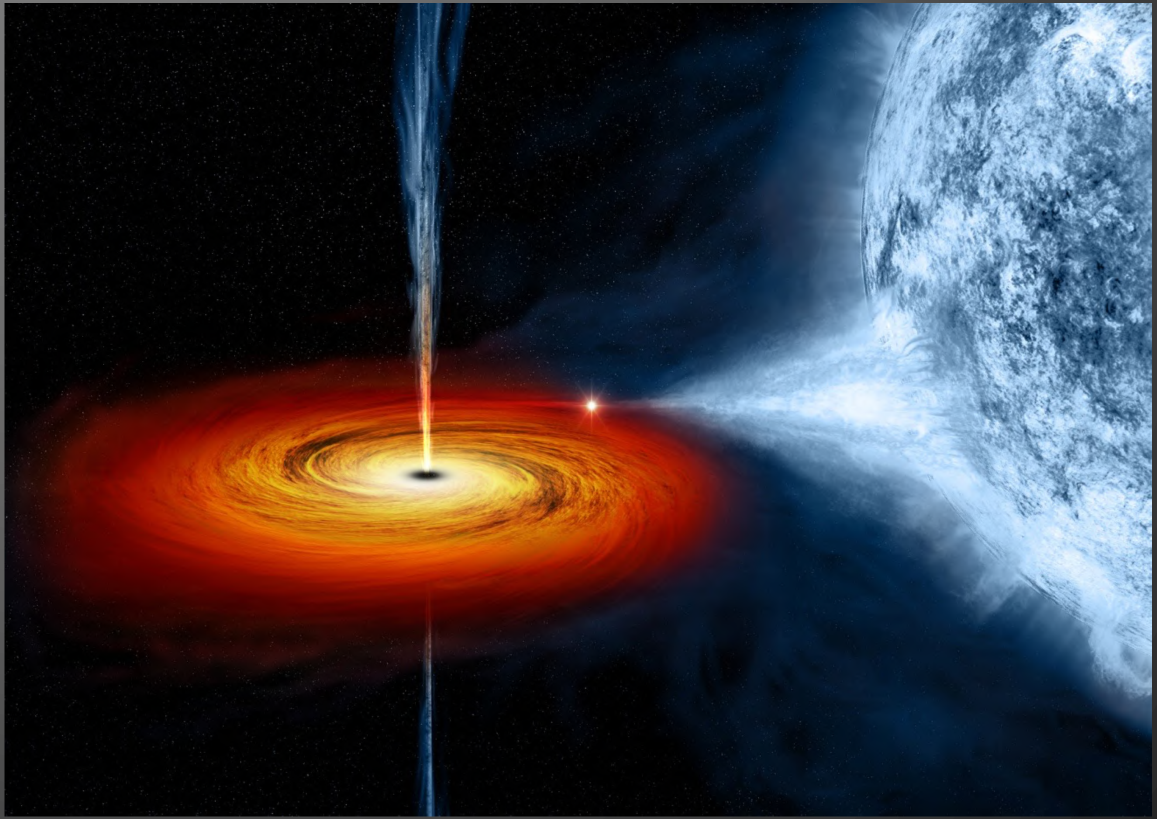
PHYSICS OF COMPACT OBJECTS

AND THEIR BINARY INTERACTIONS



AALBORG
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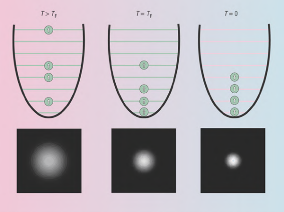
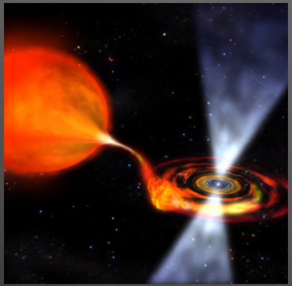
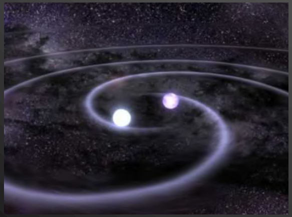
Thomas Tauris – Physics, Aalborg University

Programme







- ∗ **Introduction**
- ∗ **Degenerate Fermi Gases**
  - Non-relativistic and extreme relativistic electron / (n,p,e^-) gases
- ∗ **White Dwarfs**
  - Structure, cooling models, observations
- ∗ **Neutron Stars**
  - Structure and equation-of-state
- ∗ **Radio Pulsars**
  - Characteristics, spin evolution, magnetars, observations, timing
- ∗ **Binary Evolution and Interactions**
  - X-ray binaries, accretion, formation of millisecond pulsars, recycling
- ∗ **Black Holes**
  - Observations, characteristics and spins
- ∗ **Gravitational Waves**
  - Sources and detection, kilonovae
- ∗ **Exam**

Last lecture

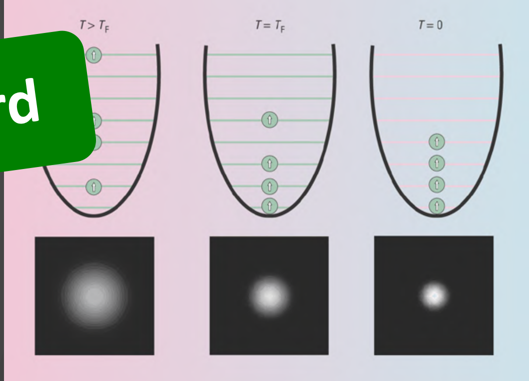
Degenerate Fermi Gases

and applications to simple EoS

- Degenerate matter
- Kinetic theory
  - Distribution function in 6-dim phase-space
  - Fermi-Dirac statistics
  - Number density (n), energy density (u), pressure (P)
- Complete degenerate ideal Fermi gas (T→ 0)
  - Non-relativistic gas
  - Relativistic gas
  - The question of relativity and degeneracy
- Polytropic EoS
  - R(M) relations for a WD (electron gas) and a NS (neutron gas)
- Three important corrections:
  - Electrostatic corrections
  - Inverse beta-decay
  - General relativity (for NSs)

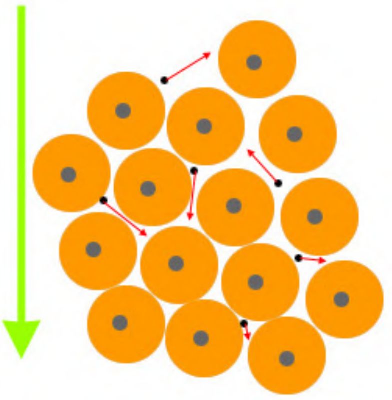
Blackboard

Blackboard




(Chandrasekhar 1931)

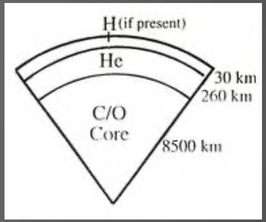
white dwarf


electrons run out of room to move around nuclei; are forced into lowest energy quantum states

White Dwarfs

Structure, EoS below neutron drip, observations

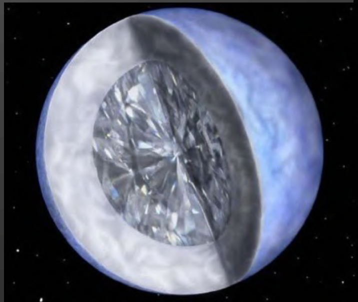
- Structure of WDs
  - Basic characteristics
  - Stability of compact objects
  - Super-Chandrasekhar mass WDs
  - Chandrasekhar mass limit

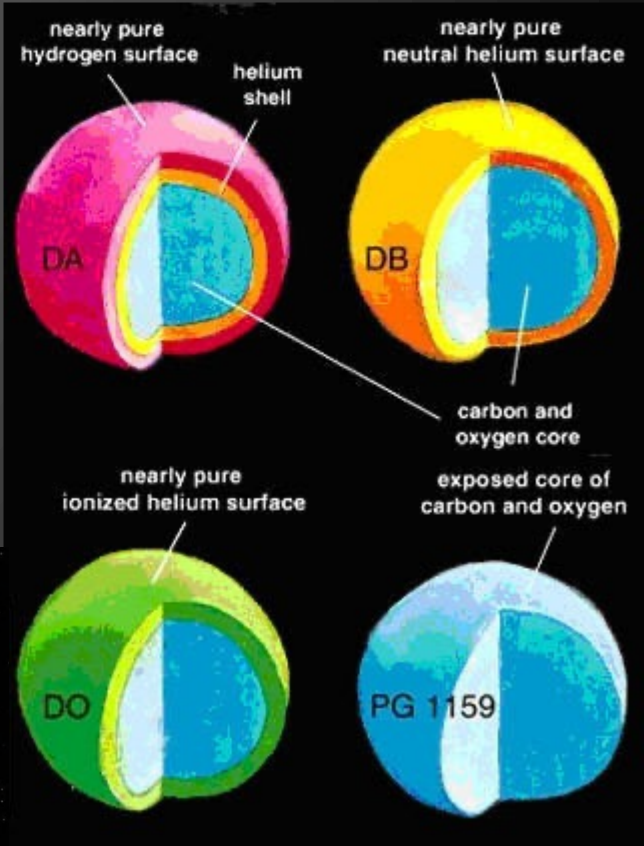


Blackboard

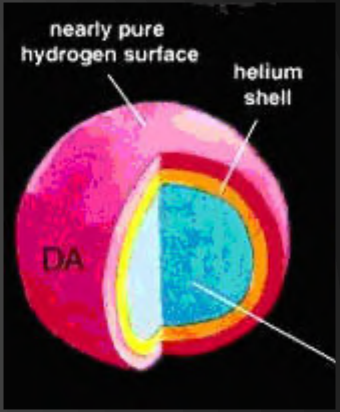
- EoS below neutron drip
  - Neutron-rich nuclei
  - Neutron drip
  - Semi-empirical mass formula
  - Including shell effects and lattice energy
  - Harrison-Wheeler EoS
  - Baym-Pethick-Sutherland (BPS) EoS

- Observations of WDs





Basic characteristics of WDs



- Typical mass: $M \approx 0.7\, M_\odot$

  Max mass: $M_{\rm Ch} \approx 1.457 \left(\frac{2}{\mu_e}\right)^2\, M_\odot$

  non-rotating WD: $M_{\rm non-rot} \approx 1.37\, M_\odot$

  max. rotating rigid WD: $M_{\rm max\ rot} \approx 1.48\, M_\odot$

- Typical radius: $R \approx 5000 - 10000\ {\rm km}$ (Earth)

- → Typical mass density: $\bar{\rho} \approx 10^6\ g\, cm^{-3}$

- Nuclear fusion has come to an end, except in very low-mass He WDs (0.14-0.18 $M_\odot$) which have thick H-envelopes (0.01 $M_\odot$).

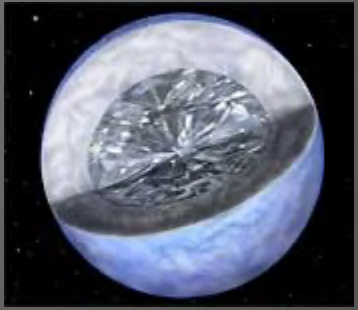
- Radiation of thermal energy of ions (WD cooling).

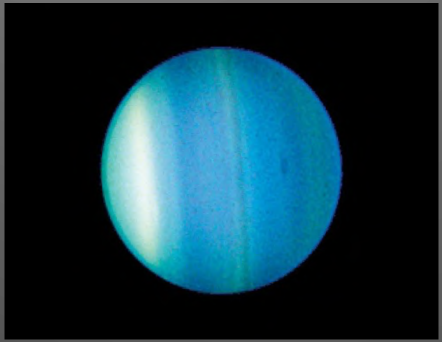
- WDs prevent gravitational collapse from pressure of degenerate electrons.

Equations-of-state below neutron drip

Applies to:

- Matter in neutron star crusts
- White dwarfs
- Gas planets





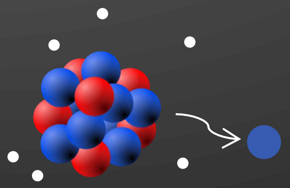
Harrison-Wheeler (HW) EoS

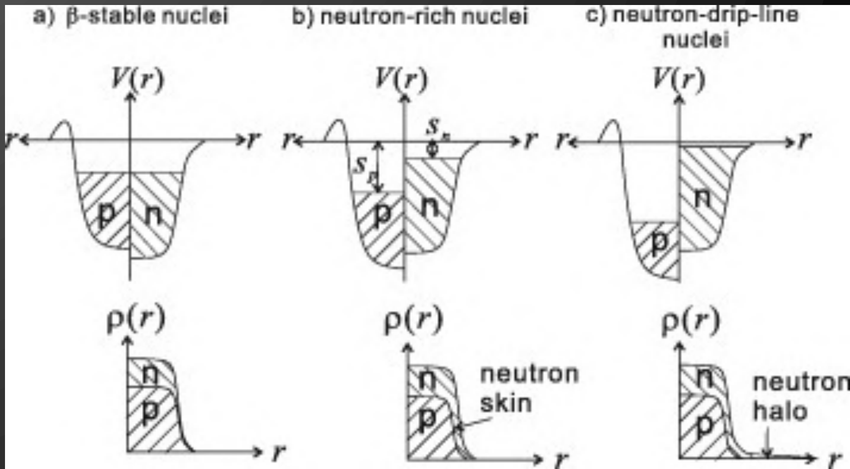
- The aim is to determine the lowest energy state for a system with 10^{57} baryons $(\approx 1\, M_{\odot})$ made of nuclei in equilibrium with a relativistic electron gas, and determine the pressure, P.
- The EoS is determined by the lowest energy state.
- ${}^{56}_{26}Fe$ nuclei have the lowest energy (competition between strong nuclear forces and Coulomb forces).
- For a system with $A \approx 10^{57}$ baryons self-gravity becomes important.
- Inverse β-decay occur in nuclei at $\rho \geq 10^{9}\ g\, cm^{-3}$ leading to more and more neutron-rich nuclei: ${}^{56}_{26}Fe \rightarrow {}^{122}_{39}Yt$ and thus Coulomb forces become weaker.
- **Neutron drip**: $\rho \geq 4 \times 10^{11}\ g\, cm^{-3}$

  Two-phase system:

  nuclei + electrons + free neutrons

  (energetically favorable for neutrons to drip out of nuclei)





Harrison-Wheeler (HW) EoS

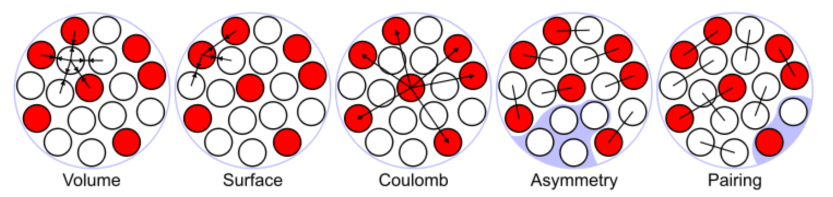
continued

- Minimizing the energy density: $\varepsilon = n_N \cdot M(A,Z) + \varepsilon_e'(n_e) + \varepsilon_n(n_n)$
  ($\varepsilon_e' = \varepsilon_e - m_e c^2$ since rest mass energy of electrons in included in $M(A,Z)$)

- Energy of nuclei via semi-empirical mass formula: (see “Moderne Fysik”)

$$M(A,Z) = \left((A-Z) m_n c^2 + Z(m_p + m_e)c^2 \ - A \cdot \bar{E}_{binding} \right)$$

liquid-drop model (volume, surface, Coulomb, asymmetry, ΔM(n,p))



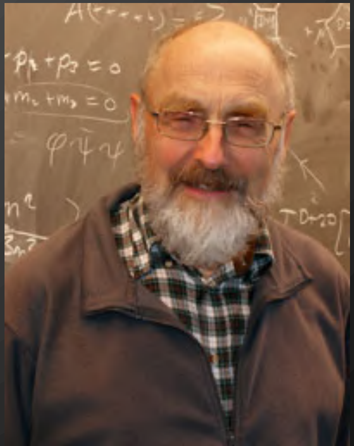
- Minimize $\varepsilon \Rightarrow \left(\frac{\partial \varepsilon}{\partial A} = \frac{\partial \varepsilon}{\partial Z} = \frac{\partial \varepsilon}{\partial \mu_e} = 0 \right)$ to obtain equilibrium and the EoS.

- Pairing and shell effects have been neglected.

- $\rho \geq 4 \times 10^{12} \ g\, cm^{-3}$ HW EoS → ideal (n,p,e^-) gas.

Baym-Pethick-Sutherland (BPS) EoS

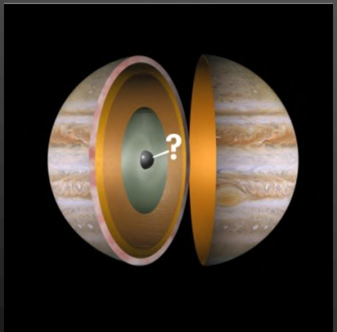
- In “real life” nuclei have integer number of A and Z.
  This affects the calculation of $M(A,Z)$
- Shell effects are included, and a better semi-empirical mass formula is used.
- Lattice energy is included: $E_{coulomb} \downarrow \quad \text{with } 15\%$
- $\varepsilon = n_N \cdot M(A,Z) + \varepsilon_e'(n_e) + \varepsilon_n(n_n) \boxed{+ \varepsilon_L}$
- $\rho \geq 4 \times 10^{11}\ g\, cm^{-3}$ (neutron drip): BPS EoS → BBP EoS
  (Baym-Bethe-Pethick EoS)

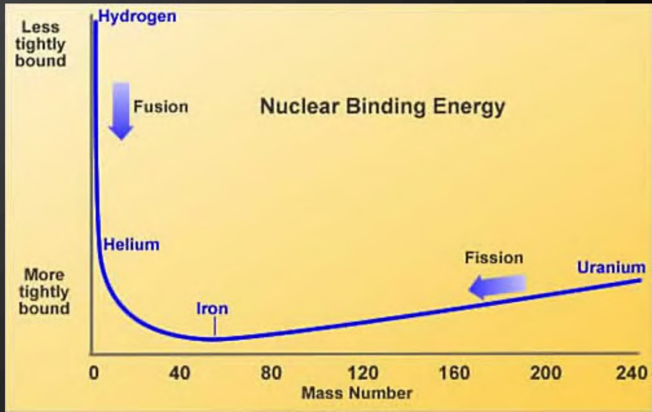


Chris Pethick
Copenhagen

Summary of cold EoS below neutron drip

- EoS for $T=0$ is fairly well known for $\rho < \rho_{drip} \quad (4\times 10^{11}\ g\,cm^{-3})$
- P is dominated by deg. electrons (which become relativistic for $\rho > 10^{7}\ g\,cm^{-3}$)
- Positive charges (protons) sit inside nuclei in ordered Coulomb lattices surrounded by an electron gas.
- $\rho < 10^{9}\ g\,cm^{-3}$: the ground state is $^{56}_{26}Fe$ (Table 2.1)
  $\rho \geq 10^{9}\ g\,cm^{-3}$: more neutron-rich nuclei …… $^{122}_{39}Yt$ (Table 3.1)
  Nuclei are stable against β-decay $n \not\rightarrow p + e^{-} + \bar{\nu}$ b/c filled Fermi sea of electrons (Pauli blocking)
- $\rho < \rho_{drip}$ describe the EoS for white dwarfs and planets.
- See Fig. 2.2 + Fig. 2.3 for an overview.





Summary of cold EoS below neutron drip

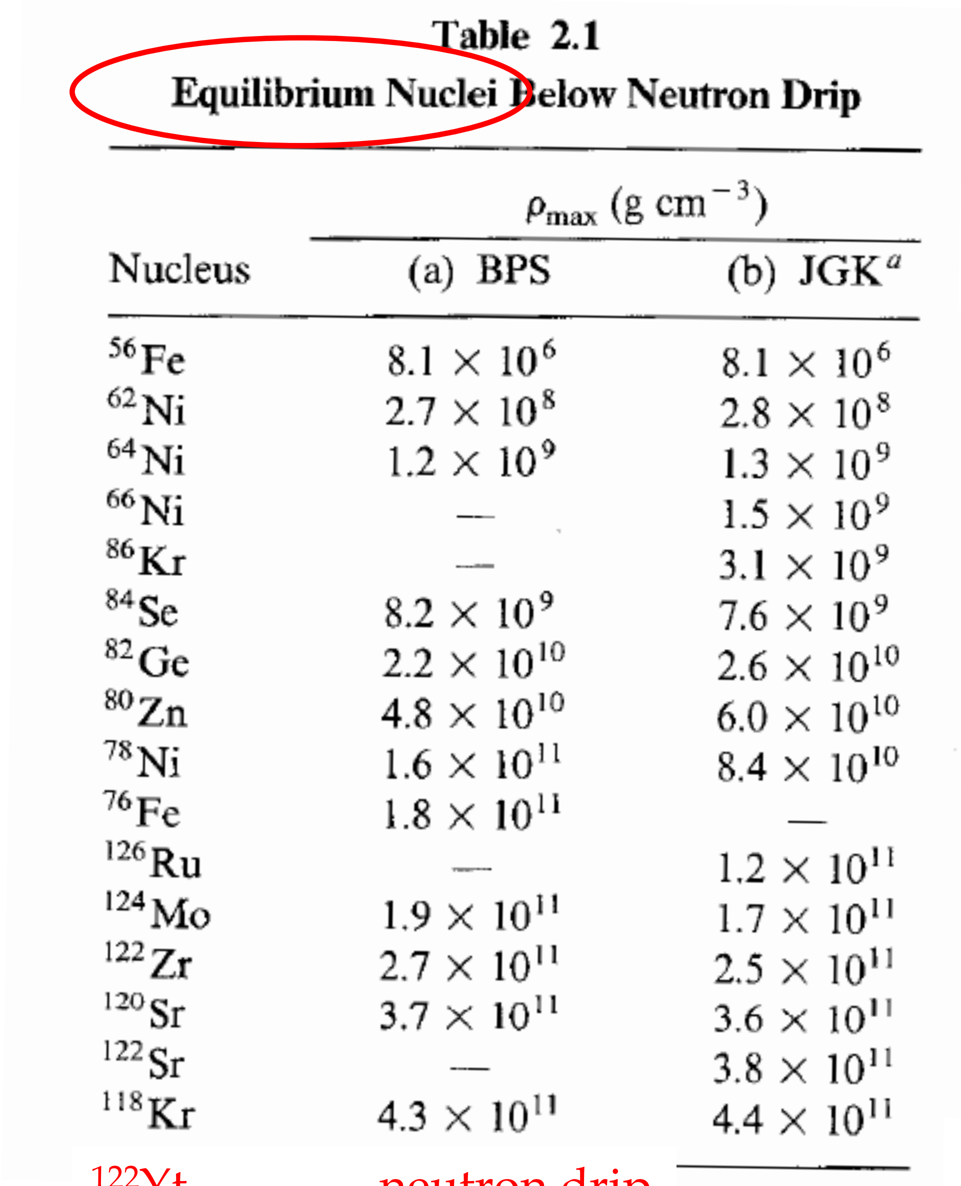


Table 2.1

Equilibrium Nuclei Below Neutron Drip

Nucleus	$\rho_{\max}$ (g cm^{-3}) (a) BPS	(b) JGK[a]
^{56}Fe	8.1×10^6	8.1×10^6
^{62}Ni	2.7×10^8	2.8×10^8
^{64}Ni	1.2×10^9	1.3×10^9
^{66}Ni	—	1.5×10^9
^{86}Kr	—	3.1×10^9
^{84}Se	8.2×10^9	7.6×10^9
^{82}Ge	2.2×10^{10}	2.6×10^{10}
^{80}Zn	4.8×10^{10}	6.0×10^{10}
^{78}Ni	1.6×10^{11}	8.4×10^{10}
^{76}Fe	1.8×10^{11}	—
^{126}Ru	—	1.2×10^{11}
^{124}Mo	1.9×10^{11}	1.7×10^{11}
^{122}Zr	2.7×10^{11}	2.5×10^{11}
^{120}Sr	3.7×10^{11}	3.6×10^{11}
^{122}Sr	—	3.8×10^{11}
^{118}Kr	4.3×10^{11}	4.4×10^{11}

^{122}Yt neutron drip

electron capture

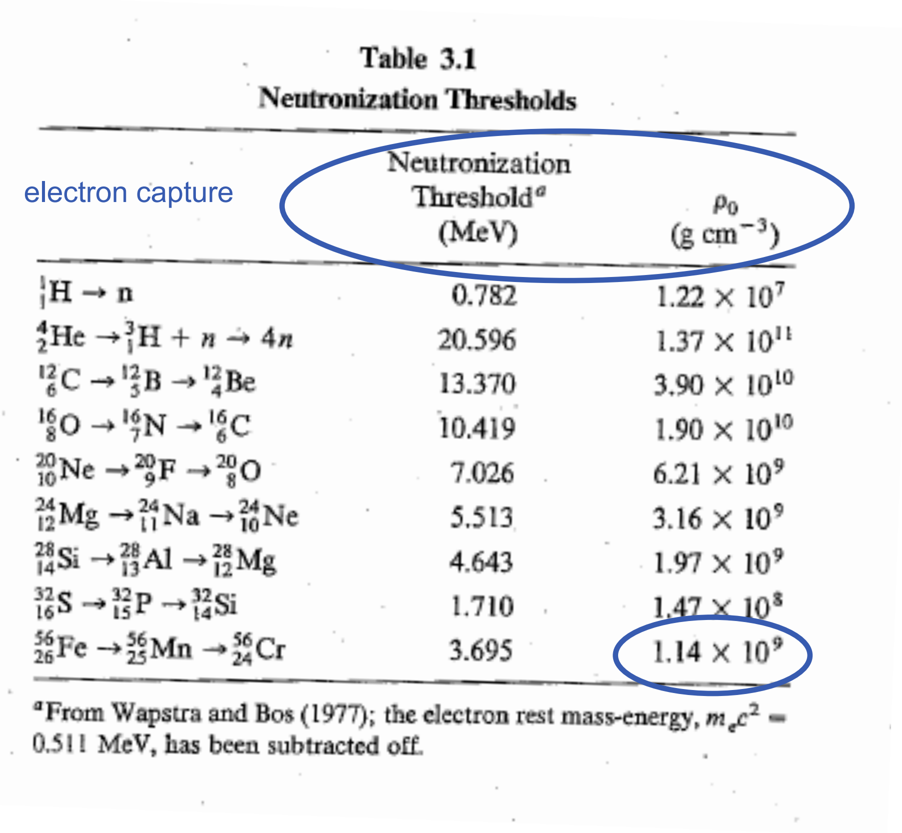


Table 3.1

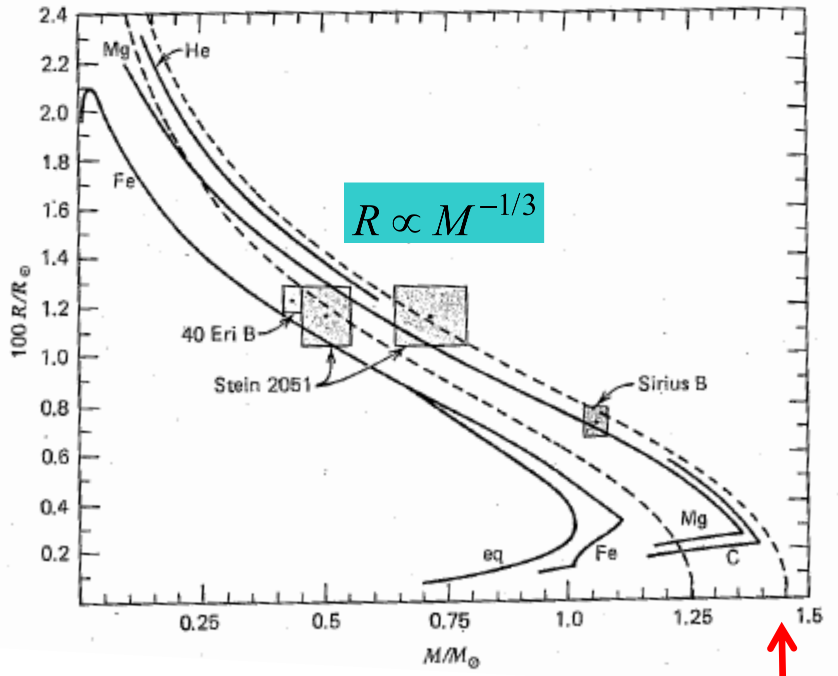
Neutronization Thresholds

	Neutronization Threshold[a] (MeV)	ρ_0 (g cm^{-3})
${}^1_1\mathrm{H} \to n$	0.782	1.22×10^7
${}^4_2\mathrm{He} \to {}^3_1\mathrm{H} + n \to 4n$	20.596	1.37×10^{11}
${}^{12}_6\mathrm{C} \to {}^{12}_5\mathrm{B} \to {}^{12}_4\mathrm{Be}$	13.370	3.90×10^{10}
${}^{16}_8\mathrm{O} \to {}^{16}_7\mathrm{N} \to {}^{16}_6\mathrm{C}$	10.419	1.90×10^{10}
${}^{20}_{10}\mathrm{Ne} \to {}^{20}_9\mathrm{F} \to {}^{20}_8\mathrm{O}$	7.026	6.21×10^9
${}^{24}_{12}\mathrm{Mg} \to {}^{24}_{11}\mathrm{Na} \to {}^{24}_{10}\mathrm{Ne}$	5.513	3.16×10^9
${}^{28}_{14}\mathrm{Si} \to {}^{28}_{13}\mathrm{Al} \to {}^{28}_{12}\mathrm{Mg}$	4.643	1.97×10^9
${}^{32}_{16}\mathrm{S} \to {}^{32}_{15}\mathrm{P} \to {}^{32}_{14}\mathrm{Si}$	1.710	1.47×10^8
${}^{56}_{26}\mathrm{Fe} \to {}^{56}_{25}\mathrm{Mn} \to {}^{56}_{24}\mathrm{Cr}$	3.695	1.14×10^9

[a] From Wapstra and Bos (1977); the electron rest mass-energy, $m_ec^2 = 0.511$ MeV, has been subtracted off.

Summary of cold EoS below neutron drip

Fig.3.1

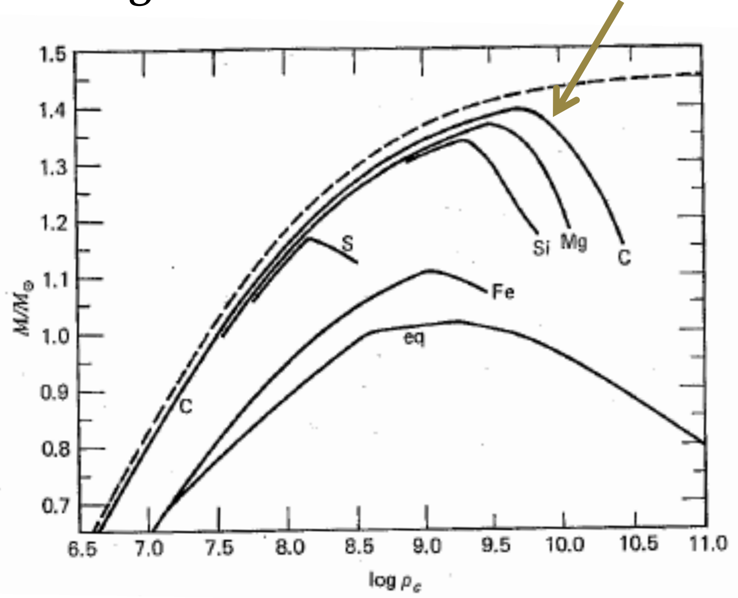


$R \propto M^{-1/3}$

$M_{Ch} = 1.457\ M_\odot$

Fig.3.2

electron capture



Summary of cold EoS below neutron drip

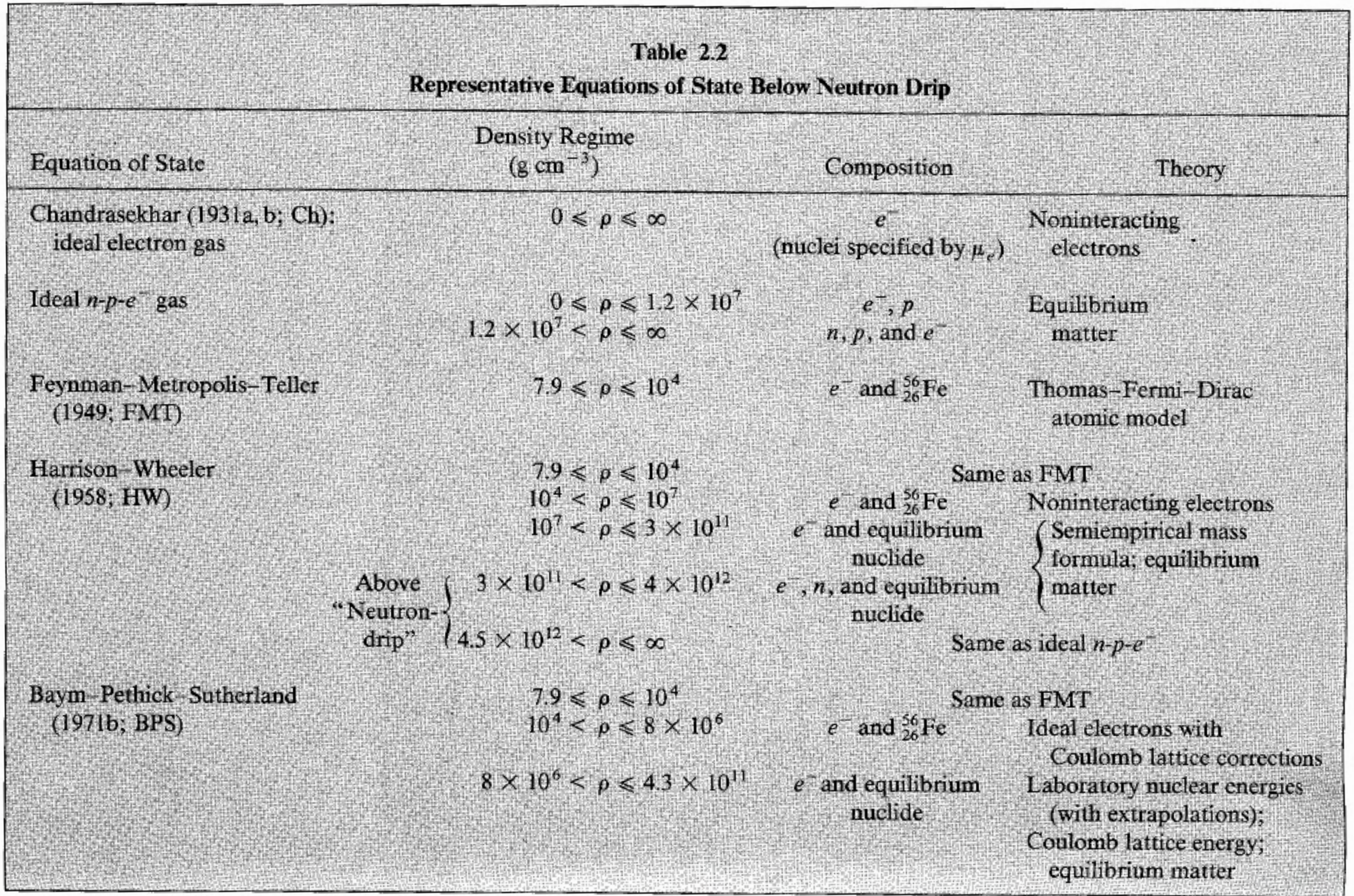


Table 2.2

Representative Equations of State Below Neutron Drip

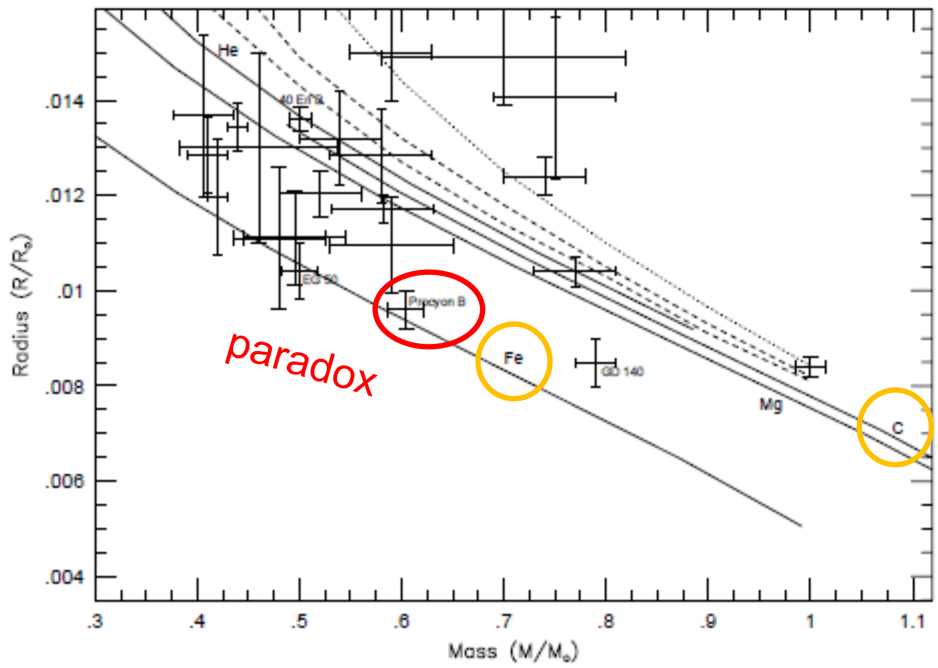
Equation of State	Density Regime (g cm^{-3})	Composition	Theory
Chandrasekhar (1931a, b; Ch): ideal electron gas	$0 \leqslant \rho \leqslant \infty$	e^- (nuclei specified by μ_e)	Noninteracting electrons
Ideal n-p-e^- gas	$0 \leqslant \rho \leqslant 1.2 \times 10^7$	e^-, p	Equilibrium matter
	$1.2 \times 10^7 < \rho \leqslant \infty$	n, p, and e^-	
Feynman–Metropolis–Teller (1949; FMT)	$7.9 \leqslant \rho \leqslant 10^4$	e^- and $^{56}_{26}\mathrm{Fe}$	Thomas–Fermi–Dirac atomic model
Harrison–Wheeler (1958; HW)	$7.9 \leqslant \rho \leqslant 10^4$	Same as FMT	
	$10^4 < \rho \leqslant 10^7$	e^- and $^{56}_{26}\mathrm{Fe}$	Noninteracting electrons
	$10^7 < \rho \leqslant 3 \times 10^{11}$	e^- and equilibrium nuclide	Semiempirical mass formula; equilibrium matter
	Above “Neutron-drip”: $3 \times 10^{11} < \rho \leqslant 4 \times 10^{12}$	e^-, n, and equilibrium nuclide	
	$4.5 \times 10^{12} < \rho \leqslant \infty$	Same as ideal n-p-e^-	
Baym–Pethick–Sutherland (1971b; BPS)	$7.9 \leqslant \rho \leqslant 10^4$	Same as FMT	
	$10^4 < \rho \leqslant 8 \times 10^6$	e^- and $^{56}_{26}\mathrm{Fe}$	Ideal electrons with Coulomb lattice corrections
	$8 \times 10^6 < \rho \leqslant 4.3 \times 10^{11}$	e^- and equilibrium nuclide	Laboratory nuclear energies (with extrapolations); Coulomb lattice energy; equilibrium matter

Derivation of the Chandrasekhar mass limit

Blackboard

Observations of WDs

testing M-R relations



paradox

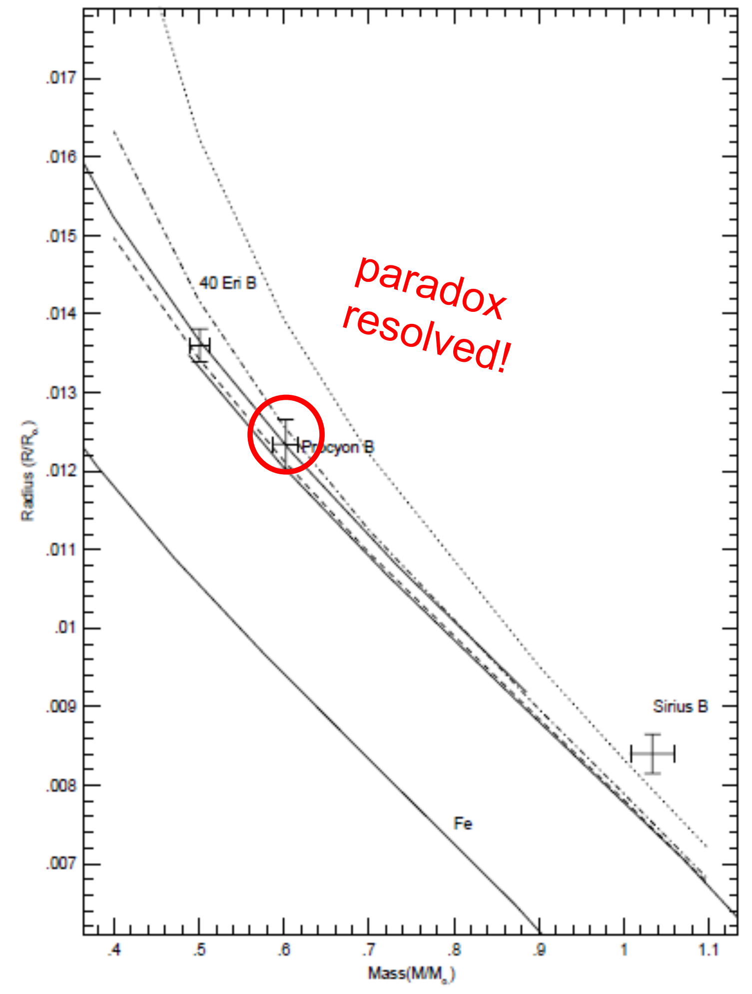
FIG. 3.—Observational support for the white dwarf mass-radius relation, showing the positions of the visual binaries, common proper-motion systems, and field white dwarfs. The field white dwarf masses were derived using published surface gravity measurements and radii based on *Hipparcos* parallaxes.

Provencal et al. (1998), ApJ 494, 759

Observations suggests Procyon B has an iron core, but in stellar evolution models WD progenitors have no silicon burning.

Observations of WDs

testing M-R relations

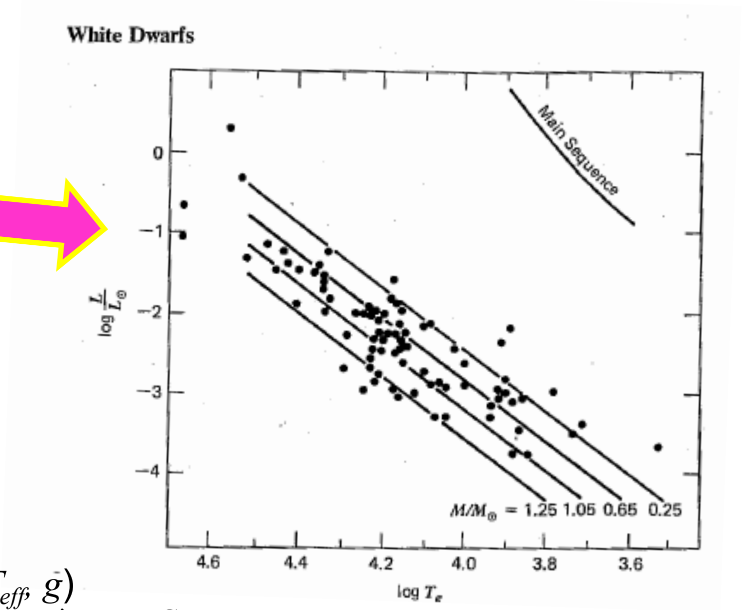


Provencal et al. (2002), ApJ 568, 324

Paradox resolved from new observations.

Observations of WDs

testing M-R relations



WDs with the same mass, M have the same radius, R

⇩

Straight line in HR-diagram, given $L = 4\pi R^2 \sigma T_{eff}^4$

(spread in M yields a band in the HR-diagram).

The **luminosity**, L is determined from measured flux, f if the distance, d is known:

$$L = f \cdot 4\pi d^2$$

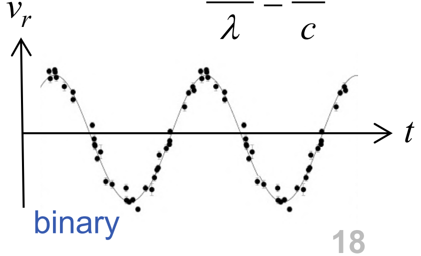
The **radius**, R is determined via a model atmosphere (T_{eff}, g) which can reproduce the observed spectrum, $f(\lambda)$

$$g = \frac{GM}{R^2}$$

The **mass**, M can be determined in binary systems via Kepler's 3rd law: $\left(\frac{2\pi}{P_{orb}}\right)^2 = \frac{GM}{a^3}$

Einstein's <u>gravitational redshift</u> also yields a M-R relation: $\boxed{\frac{\Delta\lambda}{\lambda} = \frac{GM}{Rc^2}}$

$$\frac{\Delta\lambda}{\lambda} = \frac{v_r}{c}$$

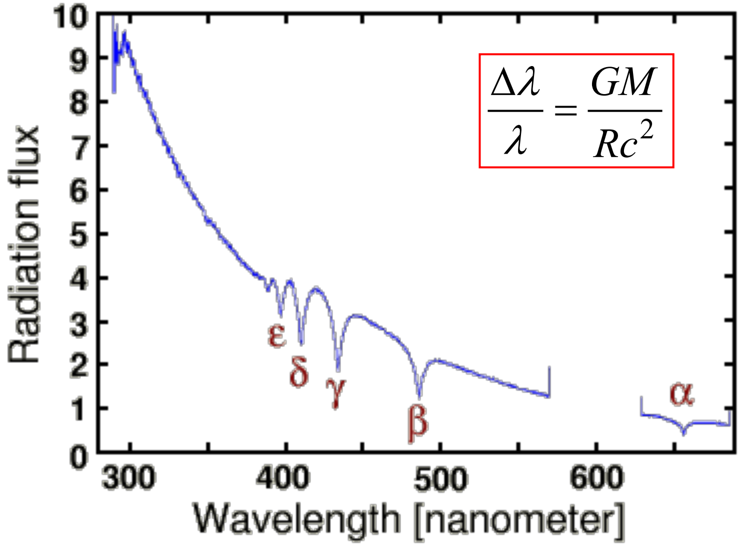


Grav. redshift can be distinguished from the usual Doppler shift caused by stellar motion in a binary via several observations at different epochs.

Observations of WDs

testing M-R relations

Gravitational redshift of Sirius B as measured by Barstow et al. (2005), MNRAS, 362, 1134
Balmer lines series:



$$\frac{\Delta\lambda}{\lambda} = \frac{GM}{Rc^2}$$

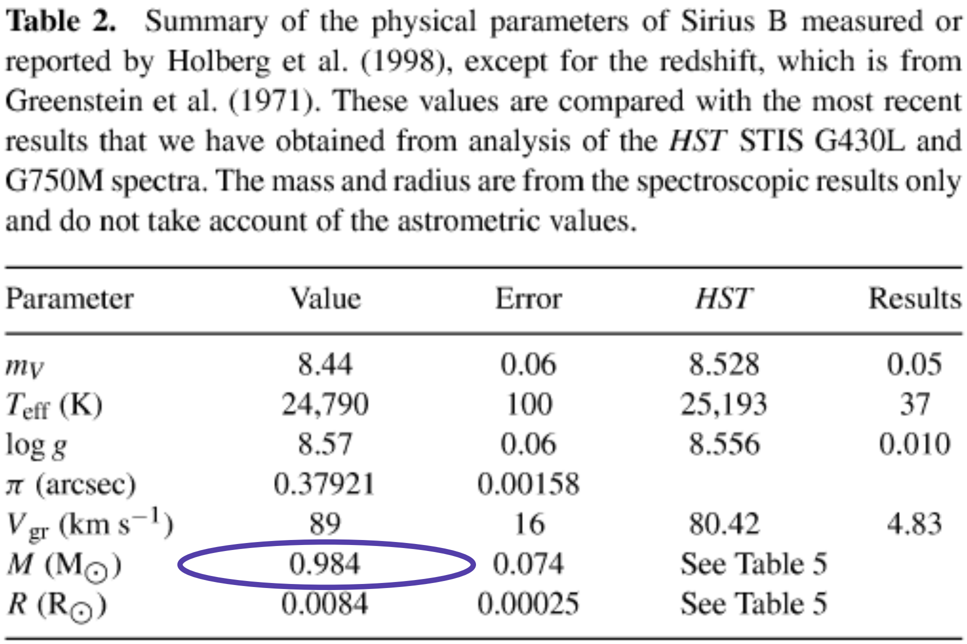


Table 2. Summary of the physical parameters of Sirius B measured or reported by Holberg et al. (1998), except for the redshift, which is from Greenstein et al. (1971). These values are compared with the most recent results that we have obtained from analysis of the *HST* STIS G430L and G750M spectra. The mass and radius are from the spectroscopic results only and do not take account of the astrometric values.

Parameter	Value	Error	*HST*	Results
m_V	8.44	0.06	8.528	0.05
$T_{\rm eff}$ (K)	24,790	100	25,193	37
$\log g$	8.57	0.06	8.556	0.010
π (arcsec)	0.37921	0.00158		
$V_{\rm gr}$ (km s^{-1})	89	16	80.42	4.83
M ($M_\odot$)	0.984	0.074	See Table 5	
R ($R_\odot$)	0.0084	0.00025	See Table 5	

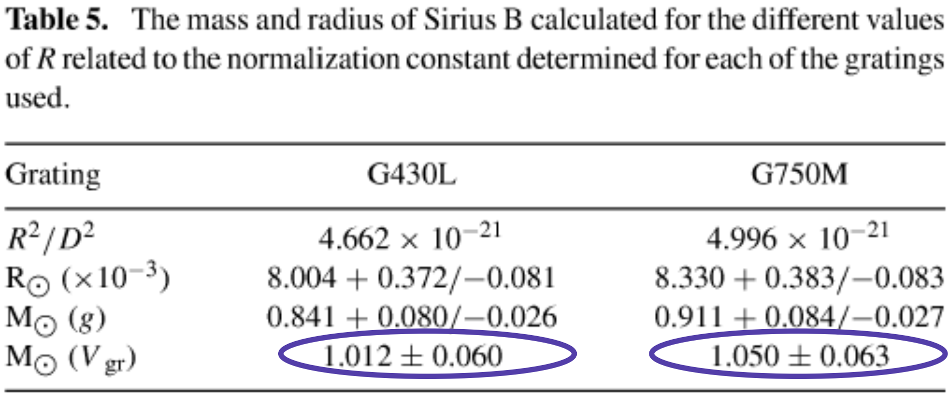


Table 5. The mass and radius of Sirius B calculated for the different values of *R* related to the normalization constant determined for each of the gratings used.

Grating	G430L	G750M
R^2/D^2	4.662×10^{-21}	4.996×10^{-21}
$R_\odot$ ($\times 10^{-3}$)	$8.004 + 0.372/-0.081$	$8.330 + 0.383/-0.083$
$M_\odot$ (g)	$0.841 + 0.080/-0.026$	$0.911 + 0.084/-0.027$
$M_\odot$ ($V_{\rm gr}$)	1.012 ± 0.060	1.050 ± 0.063

Observations of WDs

distribution in HR-diagram

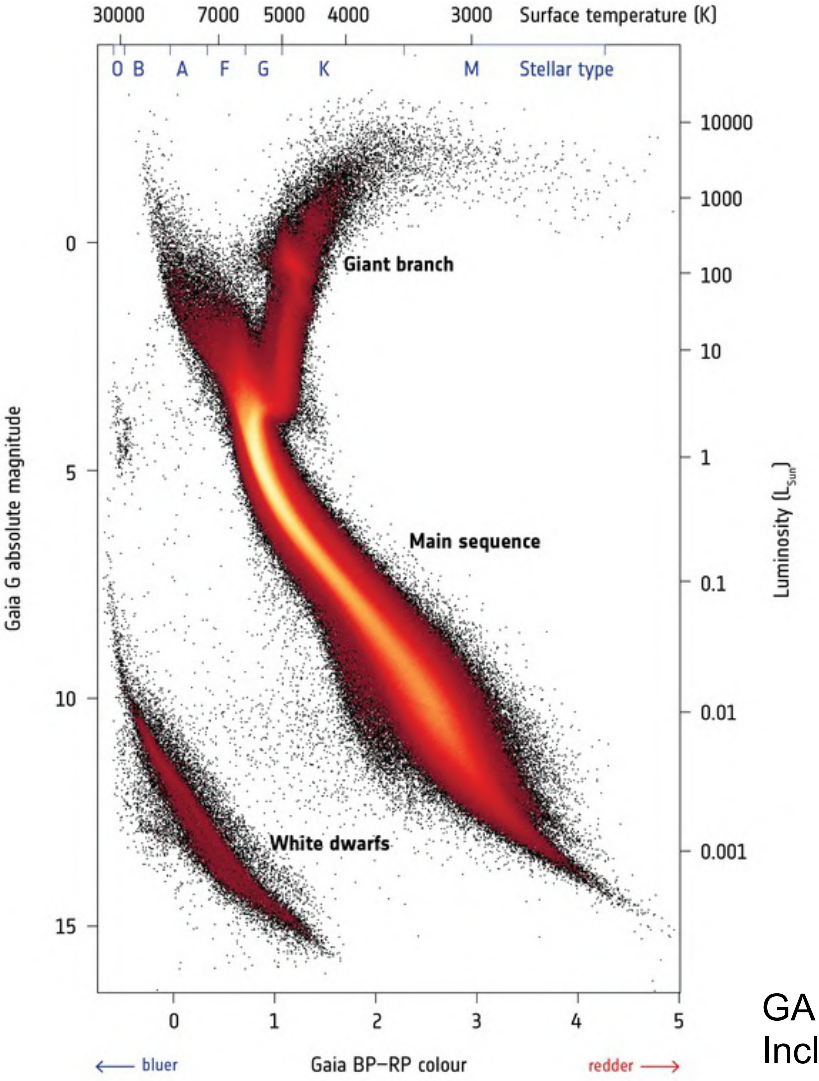
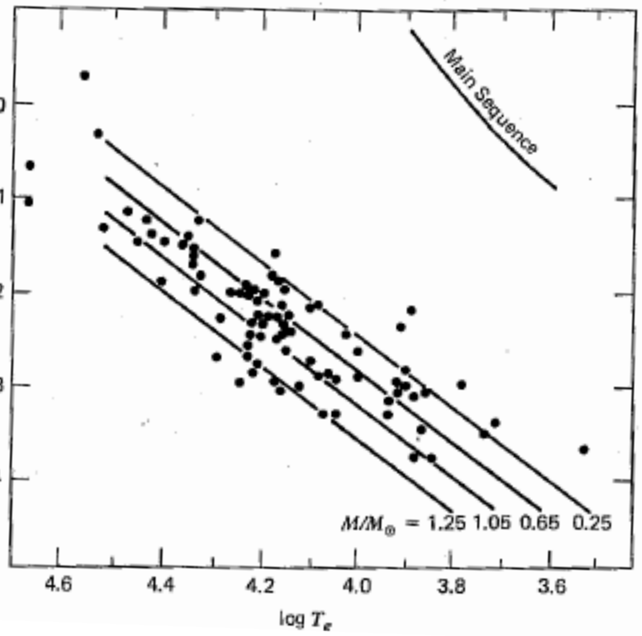
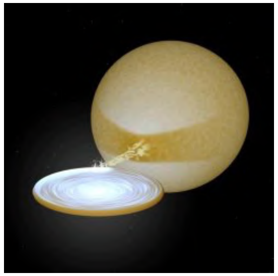


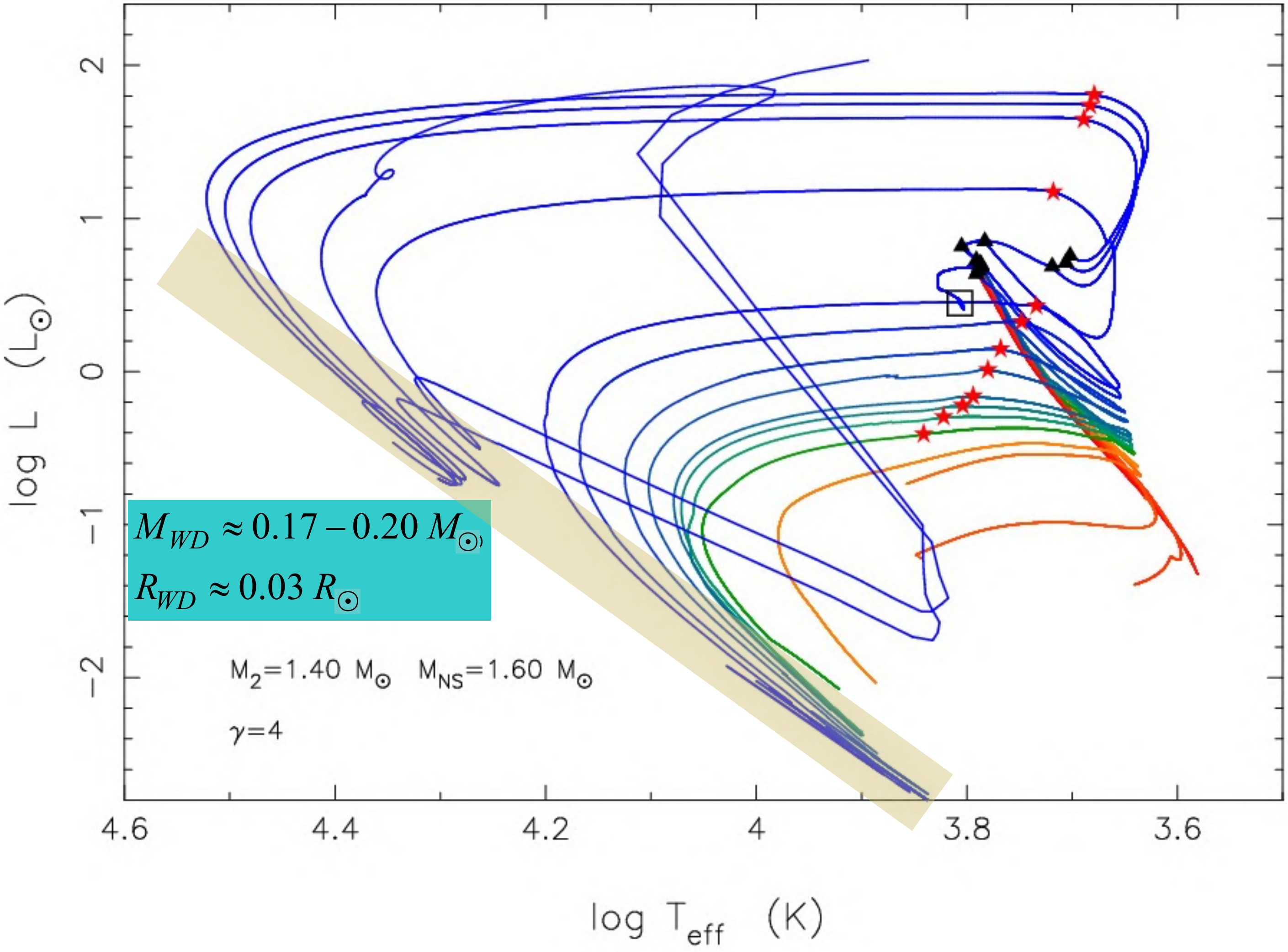
Fig.3.3



GAIA data of 4 mill. stars at distances < 5000 l.y.
Including 35.000 WDs



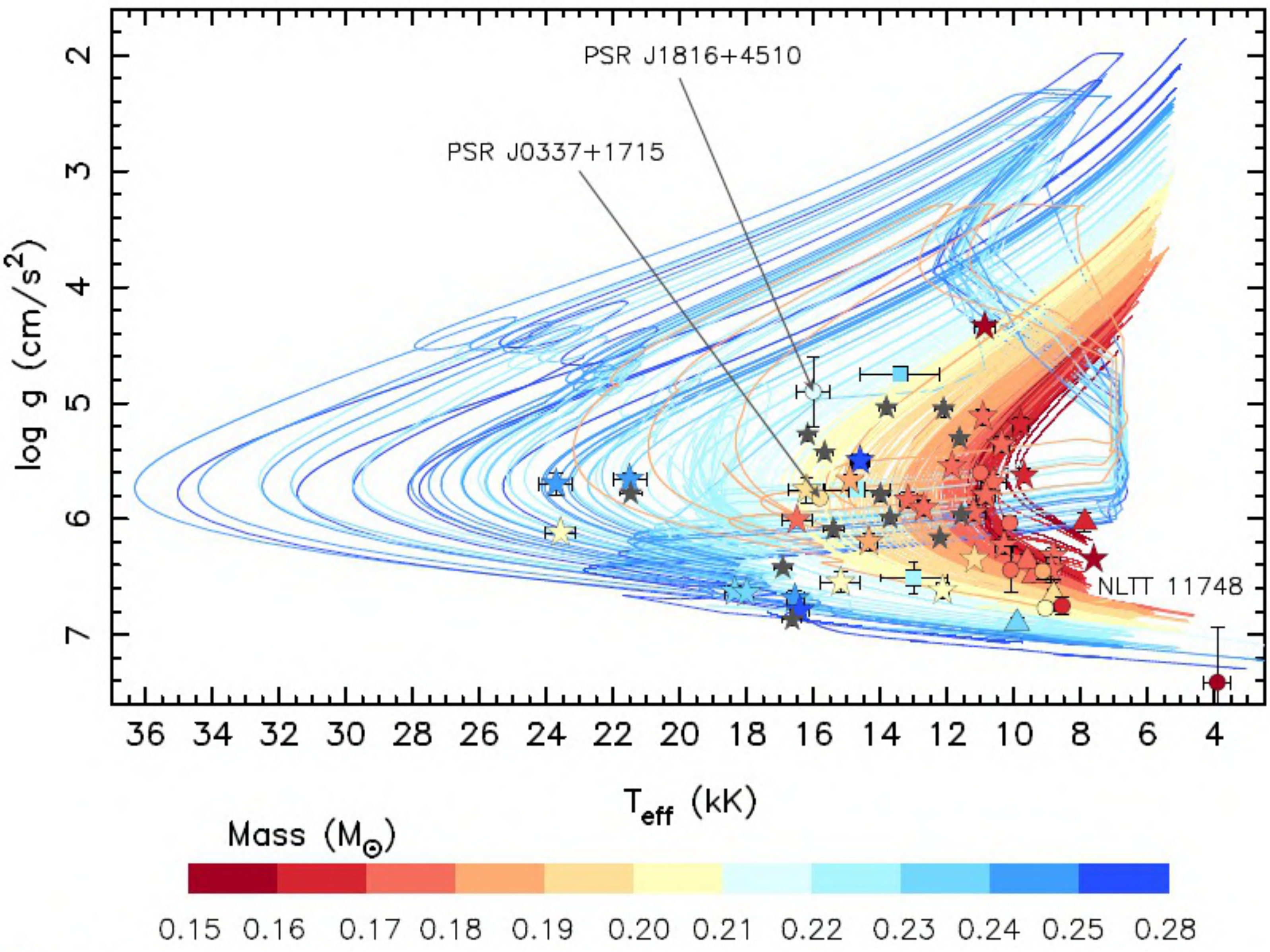
Low-mass helium WDs in binaries calculated by Alina Istrate



$M_{WD} \approx 0.17 - 0.20\ M_\odot$

$R_{WD} \approx 0.03\ R_\odot$

Istrate, Tauris, Langer & Antoniadis (2014), A&A Letters, 571, L3



Stability of Compact Objects I.

$$E_{total} = W + U$$

$$W = E_{pot}^{grav} = -\int_0^R \frac{G\,m(r)}{r}\,\overbrace{\rho\,4\pi r^2 dr}^{dm}$$

$$U = E_{\mathrm{int}} = \int_0^R \varepsilon'\,4\pi r^2 dr, \qquad \varepsilon' = \frac{P}{\Gamma - 1} \qquad \textit{energy density}$$

⇨ denoted by u in lecture 2

$$E_{total} = -\frac{3\Gamma - 4}{3(\Gamma - 1)}\,|W| = \begin{cases} -\tfrac{1}{2}\,|W| & \Gamma = 5/3 \\ 0 & \Gamma = 4/3 \end{cases}$$

virial theorem

$$\Gamma = 5/3, \quad n = 3/2 \quad non-rel.\,case \quad \rightarrow \quad E < 0 \quad stable^{\star}$$

$$\Gamma = 4/3, \quad n = 3 \quad ext-rel.\,case \quad \rightarrow \quad E = 0 \quad unstable^{\star}$$

$$\Gamma < 4/3, \quad E > 0 \quad no\ solution$$

★ against radial deformations

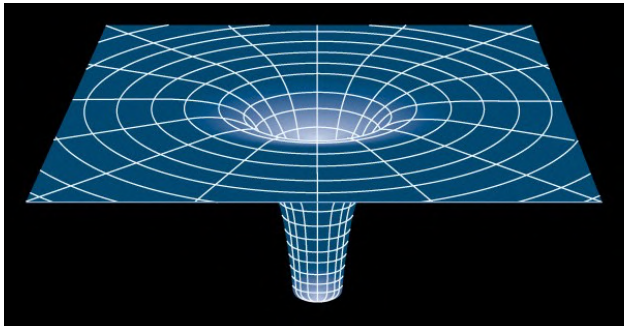
Stability of Compact Objects II.

General theory of relativity

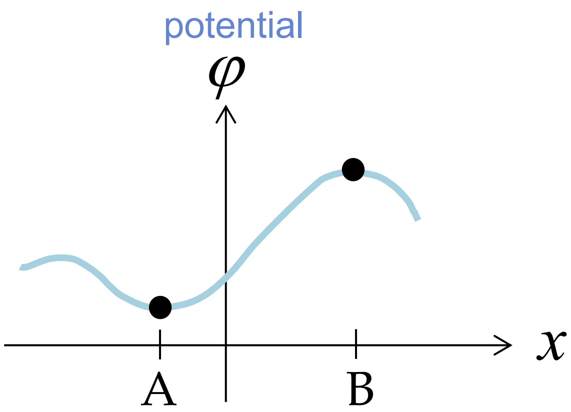
Stability criterion: $\bar{\Gamma} > \frac{4}{3} + \kappa \frac{GM}{Rc^2}$, $\bar{\Gamma} = \left. \frac{\partial \ln P}{\partial \ln \rho} \right|_S$ pressure mean of adiabatix index

GR destabilizes a star b/c gravity is 'stronger' → collapse is easier

includes rest mass *and* gravitational binding energy



Stability of Compact Objects III.



equilibrium in A and B: $\frac{\partial \varphi}{\partial x} = 0$

A: stable equilibrium $\frac{\partial^2 \varphi}{\partial x^2} > 0$

B: unstable equilibrium $\frac{\partial^2 \varphi}{\partial x^2} < 0$

Compact stars:

deviation from n=3 polytrope

$$E = \underbrace{E_{\mathrm{grav}} + E_{\mathrm{int}}}_{\text{dominating}} + \underbrace{\Delta E_{\mathrm{GR}} + \Delta E_{\mathrm{int}}}_{\text{perturbations}}$$

$\frac{\partial E}{\partial \rho_c} = 0$ *equilibrium*

$\frac{\partial^2 E}{\partial \rho_c^2} > 0$ *stable equilibrium*

⇨ $\boxed{\frac{dM}{dR} < 0}$ $\boxed{\frac{dM}{d\rho_c} > 0}$

Stability of Compact Objects IV.

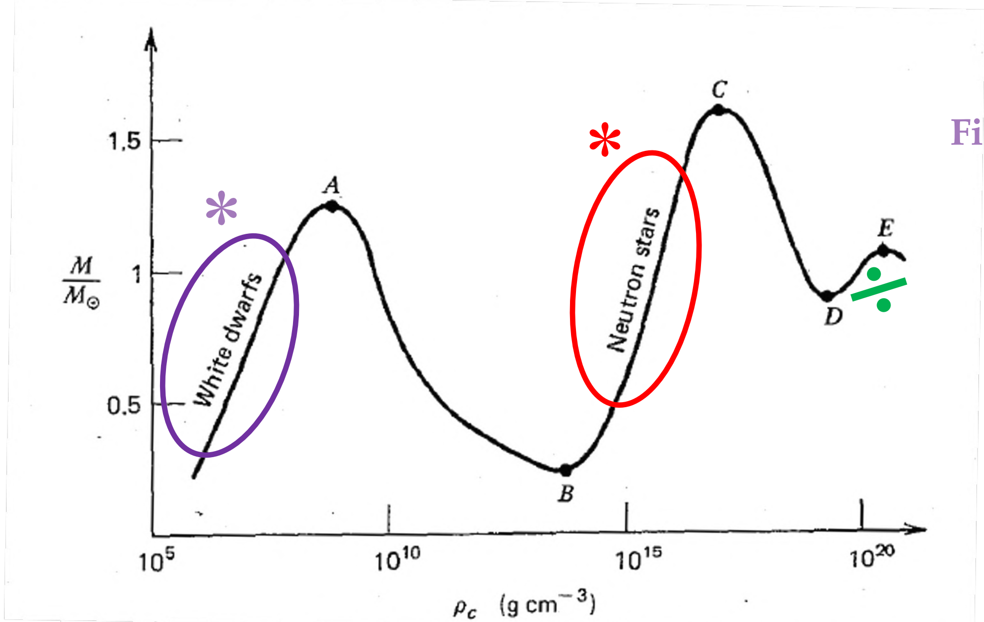


Fig.6.2

$$\frac{dM}{d\rho_c} > 0$$

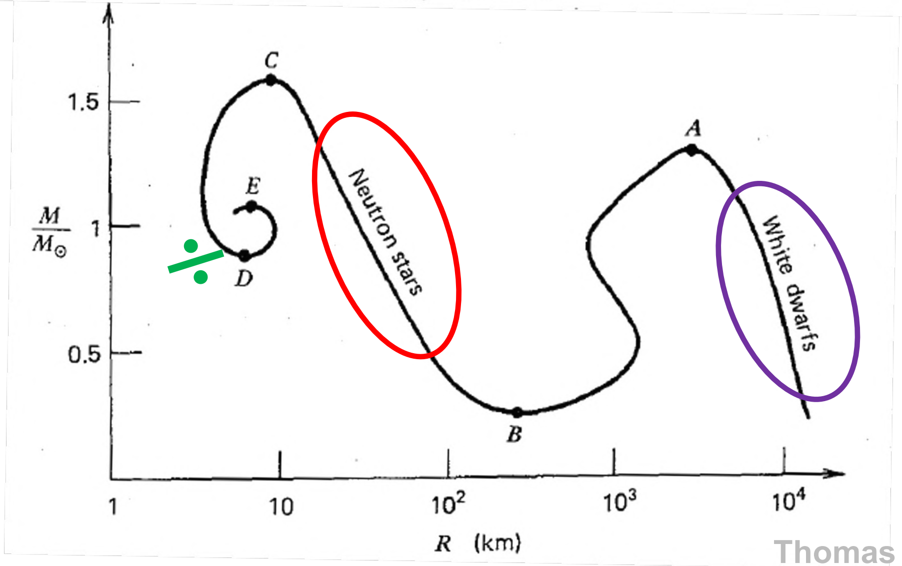


Fig.6.3

$$\frac{dM}{dR} < 0$$

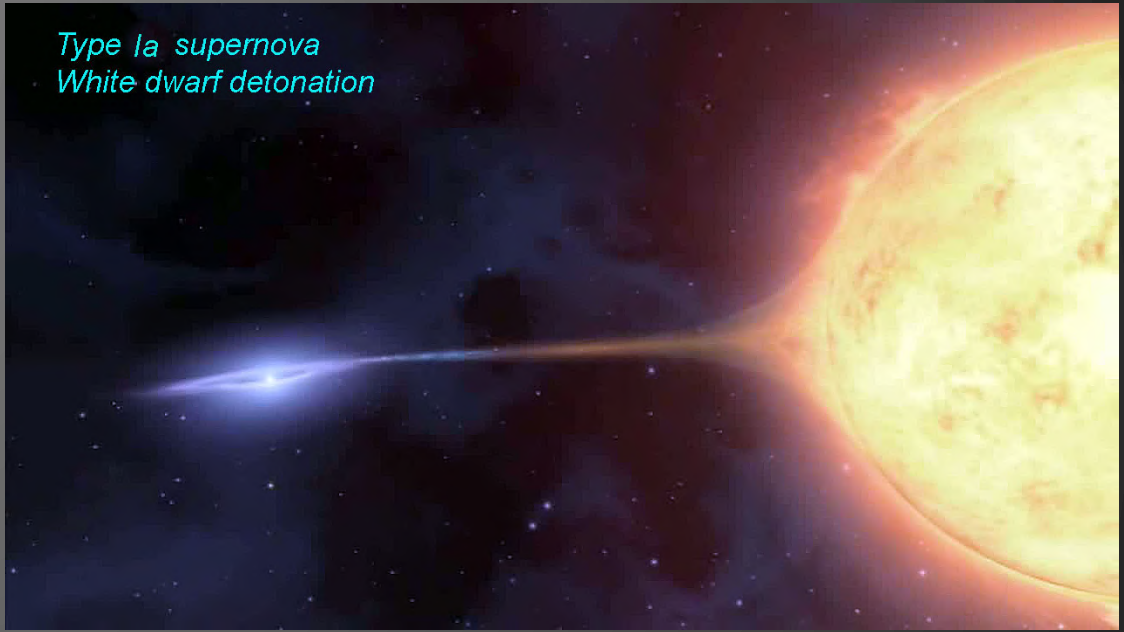
* pressure dominated by degenerate e^-

* pressure dominated by degenerate n

Super-Chandrasekhar mass WDs

Critical Chandrasekhar mass limit:

$$M_{Ch} \approx 1.457 \left(\frac{2}{\mu_e}\right)^2 M_\odot$$

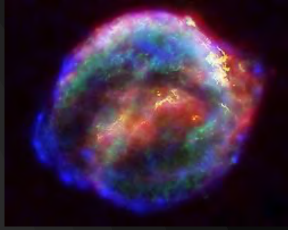


Observations of super-luminous SNe Ia suggests progenitor WDs of masses $M_{WD} > 2\,M_\odot$ (e.g. Howell et al. 2006, Scalzo et al. 2010)

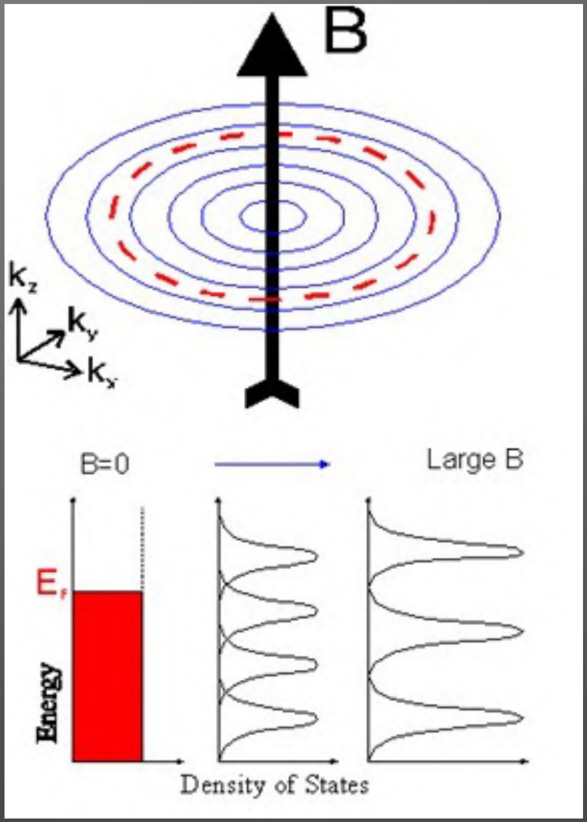
Three possible explanations for such massive WDs:

1) A merger product of two WDs (Iben & Tutukov 1984)
2) Rapid differentially rotating WD (e.g. Yoon & Langer 2005) $F_{centrifugal} \rightarrow M_{WD}^{\max} \uparrow$
   May, in principle, yield masses all the way up to $M_{WD} \approx 4\,M_\odot$
3) Strongly magnetic WDs (Das & Mukhopadhyay 2013)

Super-Chandrasekhar mass WDs

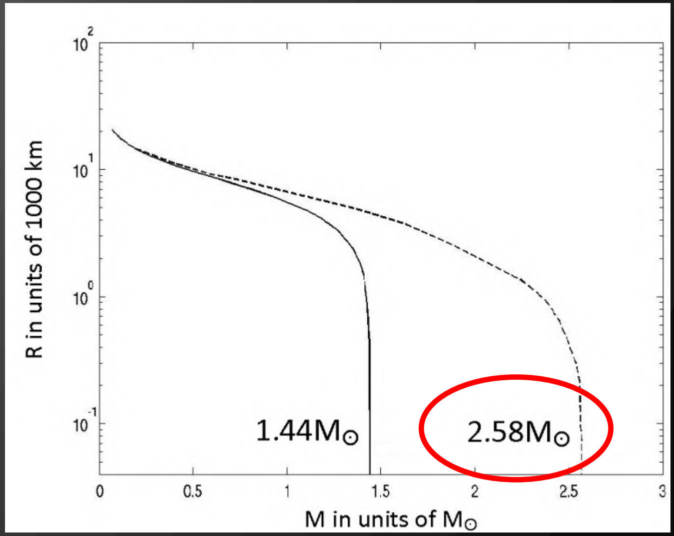


Strong B-field WDs (Das & Mukhopadhyay, 2013) $P_{magnetic} \rightarrow M_{WD}^{max} \uparrow$



$$M = \left(\frac{hc}{2G}\right)^{3/2} \frac{1}{(\mu_e m_H)^2} \approx \frac{10.312}{\mu_e^2} M_\odot,$$

Requires $B_{WD} > 4.4 \times 10^{13}\,G$ Unrealistic(?)
for Landau quantization to be efficient

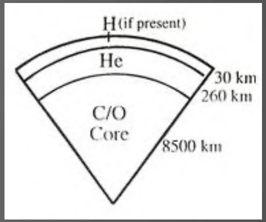


Summary

White Dwarfs

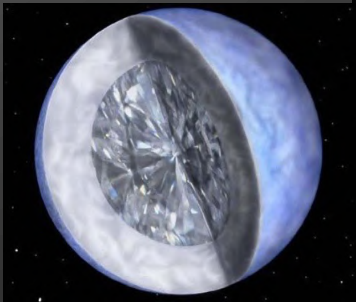
Structure, EoS below neutron drip, observations

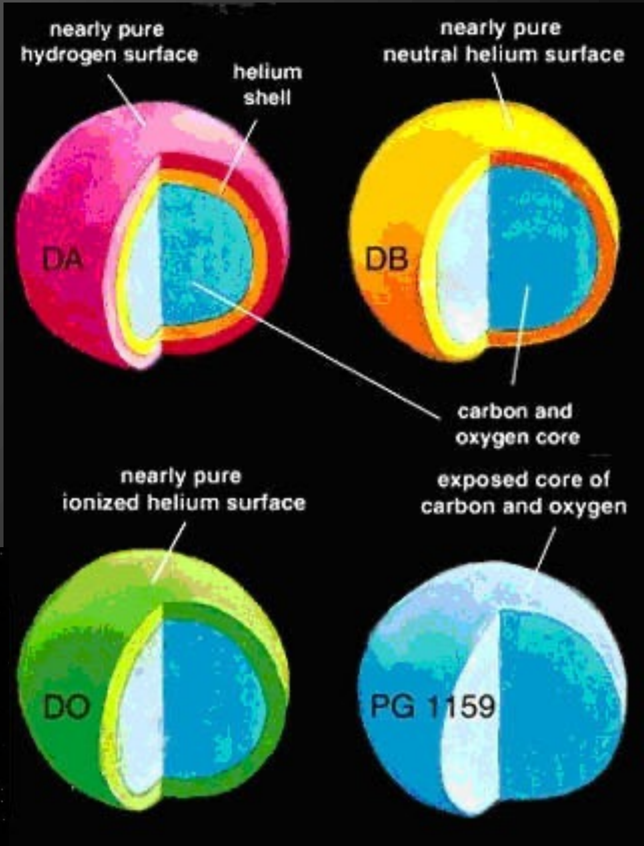
- Structure of WDs
  - Basic characteristics
  - Stability of compact objects
  - Super-Chandrasekhar mass WDs
  - Chandrasekhar mass limit



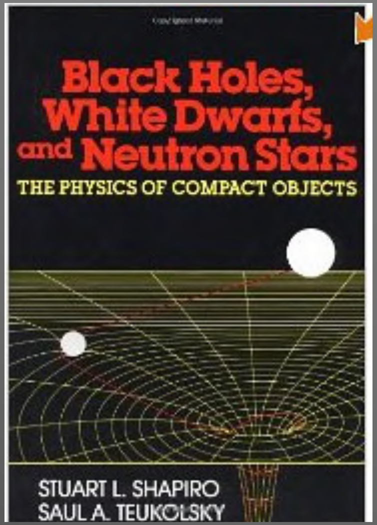
Blackboard

- EoS below neutron drip
  - Neutron-rich nuclei
  - Neutron drip
  - Semi-empirical mass formula
  - Including shell effects and lattice energy
  - Harrison-Wheeler EoS
  - Baym-Pethick-Sutherland (BPS) EoS
- Observations of WDs





Physics of Compact Objects
week 3



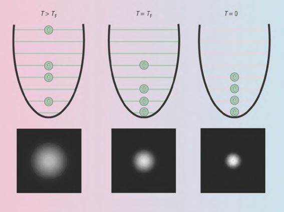
Shapiro & Teukolsky (1983), Wiley-Interscience

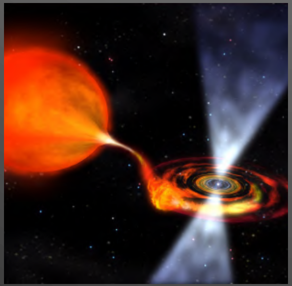
Curriculum
- Chapter 3: p.(55-57), 59, 61-72
  (Chapter 6: Figs.6.2+6.3)

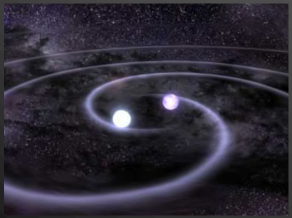
Exercises: #21
- Mon. Sep.18, 10:15-12:00

Next lecture: Cooling of White Dwarfs
S&T Chapter 4.
- Mon. Sep.25, 08:15-10:00, Aud. 2.115

Programme







- ∗ **Introduction**
- ∗ **Degenerate Fermi Gases**
  Non-relativistic and extreme relativistic electron / (n,p,e^-) gases
- ∗ **White Dwarfs**
  Structure, cooling models, observations
- ∗ **Neutron Stars**
  Structure and equation-of-state
- ∗ **Radio Pulsars**
  Characteristics, spin evolution, magnetars, observations, timing
- ∗ **Binary Evolution and Interactions**
  X-ray binaries, accretion, formation of millisecond pulsars, recycling
- ∗ **Black Holes**
  Observations, characteristics and spins
- ∗ **Gravitational Waves**
  Sources and detection, kilonovae
- ∗ **Exam**