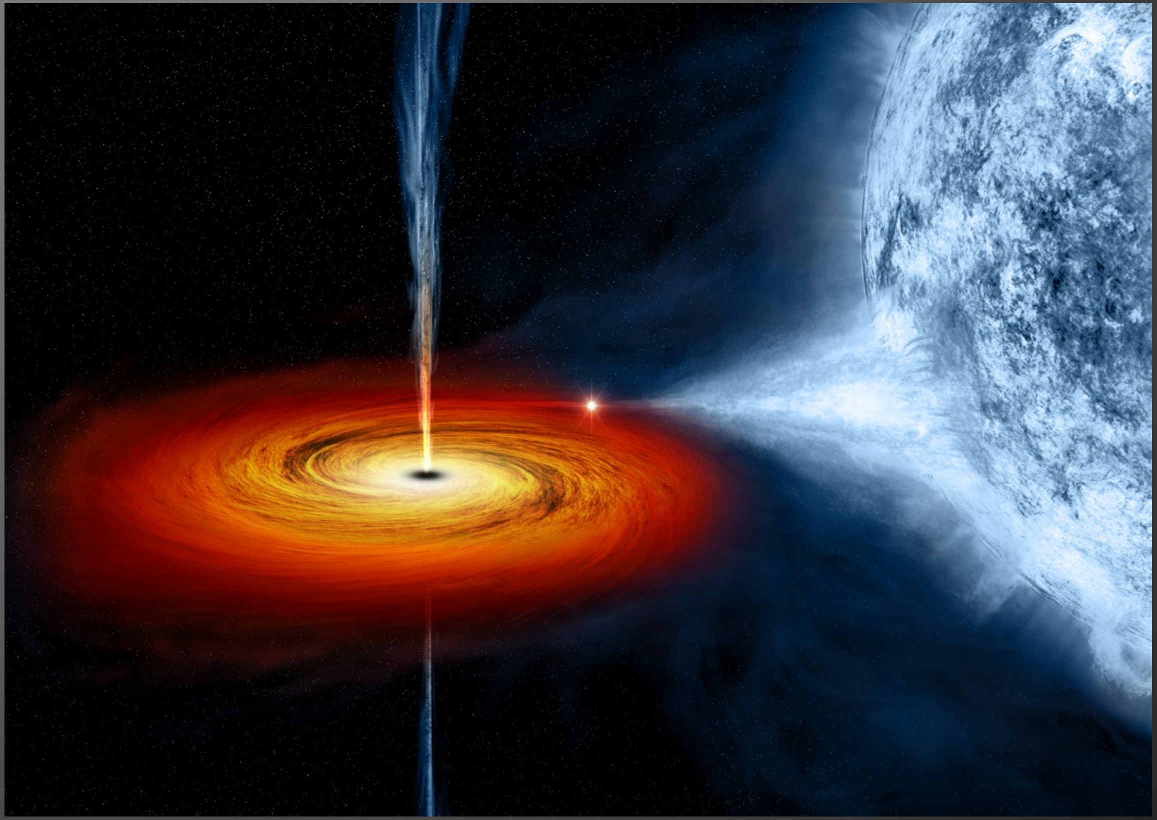
PHYSICS OF COMPACT OBJECTS

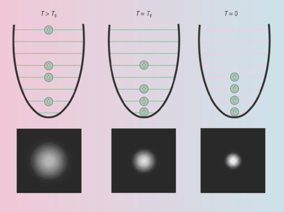
AND THEIR BINARY INTERACTIONS

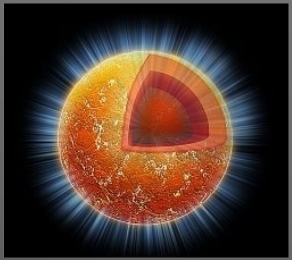


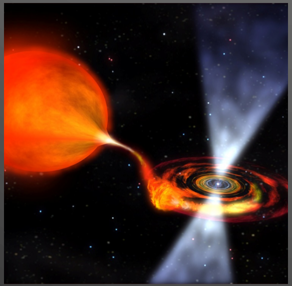
AALBORG UNIVERSITY

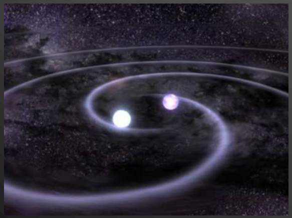
Thomas Tauris – Physics, Aalborg University

Programme









- ▣ ∗ **Introduction**
- ▣ ∗ **Degenerate Fermi Gases**

  Non-relativistic and extreme relativistic electron / (n,p,e^-) gases
- ▣ ∗ **White Dwarfs**

  Structure, cooling models, observations
- ▣ ∗ **Neutron Stars**

  Structure and equation-of-state
- ▣ ∗ **Radio Pulsars**

  Characteristics, spin evolution, magnetars, observations, timing
- ▣ ∗ **Binary Evolution and Interactions**

  X-ray binaries, accretion, formation of millisecond pulsars, recycling
- ▣ ∗ **Black Holes**

  Observations, characteristics and spins
- ▣ ∗ **Gravitational Waves**

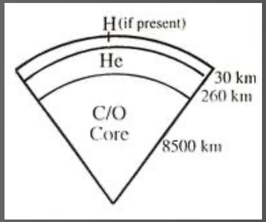
  Sources and detection, kilonovae
- ▣ ∗ **Exam**

Last week

White Dwarfs

Structure, EoS below neutron drip, observations

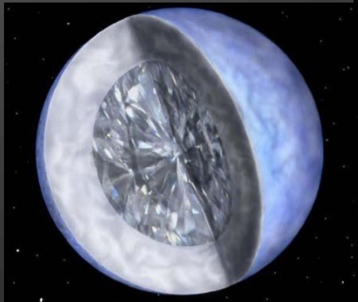
- Structure of WDs
  - Basic characteristics
  - Stability of compact objects
  - Super-Chandrasekhar mass WDs
  - Chandrasekhar mass limit

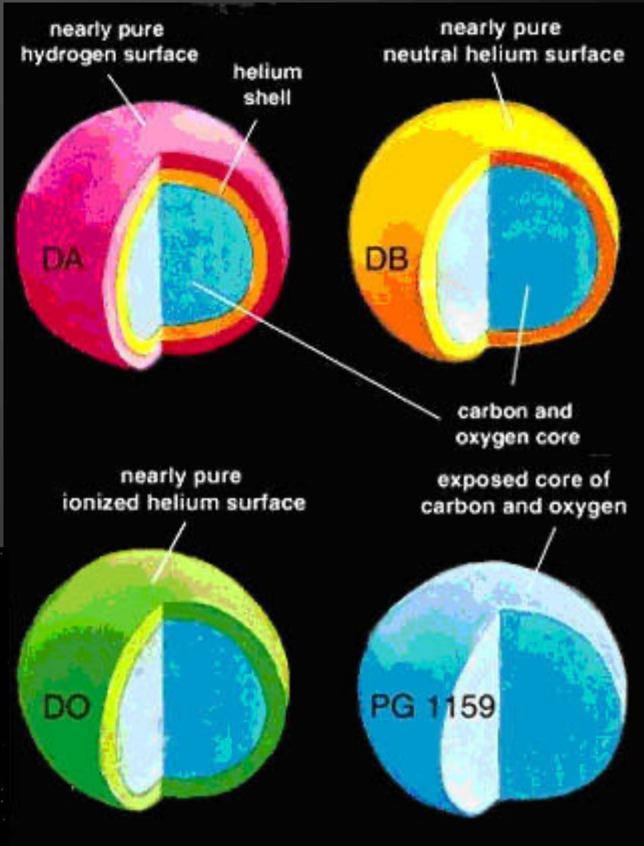


Blackboard

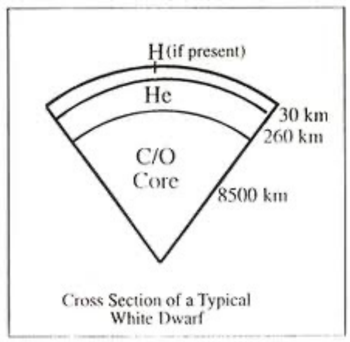
- EoS below neutron drip
  - Neutron-rich nuclei
  - Neutron drip
  - Semi-empirical mass formula
  - Including shell effects and lattice energy
  - Harrison-Wheeler EoS
  - Baym-Pethick-Sutherland (BPS) EoS

- Observations of WDs





Cooling of White Dwarfs



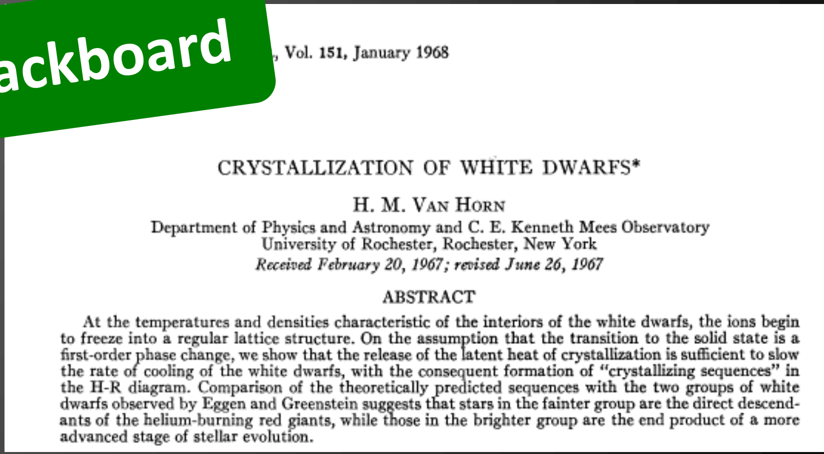
- Surface layers and core structure
- Photon diffusion equation

$$L = -4\pi r^2 \frac{c}{3\kappa\rho} \frac{d}{dr}\left(aT^4\right)$$

  - Temperature gradient
  - Pressure gradient (via hydrostatic equilibrium)
  - Core-surface boundary conditions
  - Luminosity as a function of (M, T_*)

Blackboard

- Elementary treatment of WD cooling
  - Residual ion thermal energy
  - Cooling age
- Crystallization
  - Rapid cooling
- Observational support of WD cooling models



, Vol. 151, January 1968

CRYSTALLIZATION OF WHITE DWARFS*

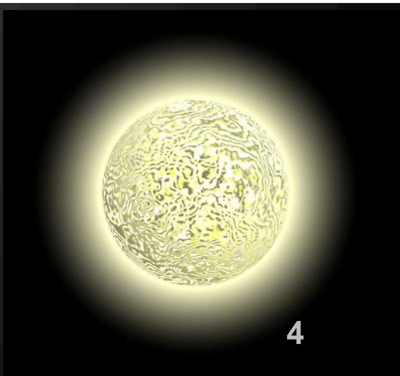
H. M. VAN HORN

Department of Physics and Astronomy and C. E. Kenneth Mees Observatory
University of Rochester, Rochester, New York

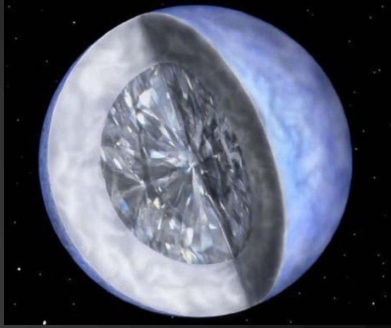
Received February 20, 1967; revised June 26, 1967

ABSTRACT

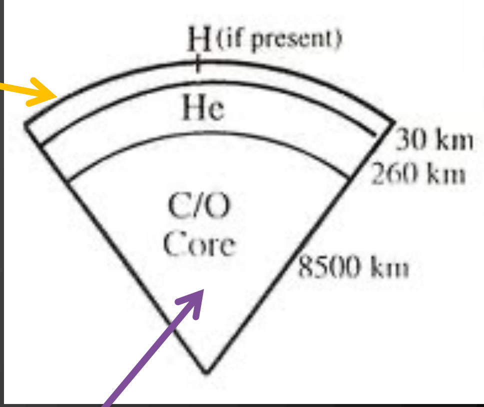
At the temperatures and densities characteristic of the interiors of the white dwarfs, the ions begin to freeze into a regular lattice structure. On the assumption that the transition to the solid state is a first-order phase change, we show that the release of the latent heat of crystallization is sufficient to slow the rate of cooling of the white dwarfs, with the consequent formation of "crystallizing sequences" in the H-R diagram. Comparison of the theoretically predicted sequences with the two groups of white dwarfs observed by Eggen and Greenstein suggests that stars in the fainter group are the direct descendants of the helium-burning red giants, while those in the brighter group are the end product of a more advanced stage of stellar evolution.



Cooling of White Dwarfs



Structure of WDs

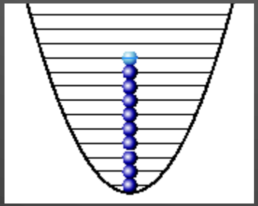


Surface layers: (H) He

non-degenerate layers in "radiative equilibrium"
LTE with outward energy flux by *diffusion of photons*
(only small gradient in net flux, I_ν: Planck function)

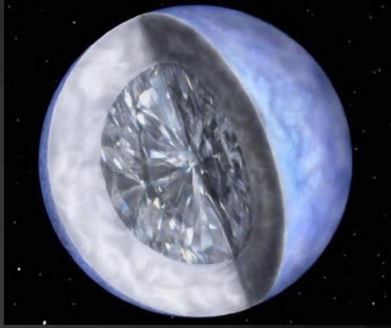
Interior: CO, ONeMg (He WDs in close binaries)

electrons are completely degenerate
→ electrons have a large mean free path, λ_e
because of the filled Fermi sea:



→ thermal conductivity, τ_{th} is large
→ temperature, T is uniform (isothermal core)

Cooling of White Dwarfs



- Photon diffusion equation: $L = -4\pi r^2 \frac{c}{3\kappa\rho}\frac{d}{dr}\left(aT^4\right)$
  - Temperature gradient
  - Pressure gradient (via hydrostatic equilibrium)
  - Core-surface boundary conditions
  - Luminosity as a function of (M, T_*)

$$L = -4\pi r^2 \frac{c}{3\kappa\rho}\frac{d}{dr}\left(aT^4\right) \quad \Leftrightarrow \quad \frac{dT}{dr} = ...$$

Ex.23

$$\wedge \quad \frac{dP}{dr} = -g\rho = -G\frac{m(r)}{r^2}\rho \qquad \text{hydrostatic equil.}$$

b-f photoionization of atoms
f-f inverse bremstrahlung of e^-

$$\Updownarrow$$

$$\frac{dP}{dT} = ... \quad \wedge \quad \kappa = \kappa_0 \rho T^{-3.5} \qquad \text{Kramer's opacity} \quad \left(I(x) = I_0 e^{-x/\lambda}, \qquad \lambda = \frac{1}{\kappa\rho} \right)$$

mean free path

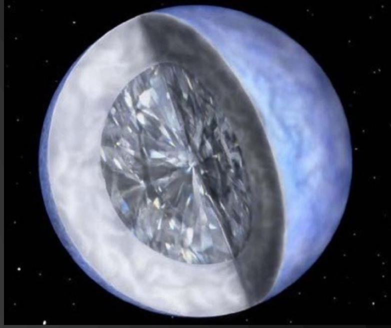
$$\Downarrow \quad \wedge \quad P = \frac{\rho}{\mu m_u}kT \qquad \text{ideal gas}$$

Integrate (from surface and inwards)…*

$$(1) \quad P \propto \sqrt{\frac{M}{L}}\,T^{17/4}$$

* Boundary conditions at surface: $P = 0, \quad T = 0$
$m(r) = M$ (thin envelope)

Cooling of White Dwarfs



Core boundary condition
(transition from surface layers to core region) at density: ρ_*

$$P_{gas} = P_{\deg} \quad \Leftrightarrow \quad \frac{\rho_*}{\mu m_u} kT_* = K\rho_*^{5/3} \quad \rightarrow \quad \rho_*^{5/3} \propto T_*^{5/2}$$

insert

$$P_{gas} \propto \sqrt{\frac{M}{L}}\, T_*^{17/4} = K\rho_*^{5/3}$$

$$\Updownarrow$$

$$(2) \quad \boxed{L \propto M\, T_*^{3.5}} \qquad (L = C \cdot M \cdot T_*^{3.5})$$

Typically: $T_* \approx 10^6 - 10^7\ K \quad \Rightarrow \quad L = 10^{-5} - 10^{-2}\ L_\odot$

core temperature

Note

$$\rho_* < 10^3\ \mathrm{g\,cm^{-3}} \ll \rho_c$$

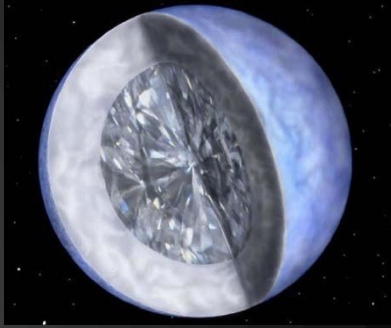
$$\Downarrow$$

$$r_{\text{surface layers}} \ll R_{\text{WD}}$$

Ex.24c

Thus the assumption of a fully degenerate star (cold EoS) is valid!
(M,R)-relations obtained earlier are ok!

Cooling of White Dwarfs



Loss of residual thermal energy of ions → radiation

$$E_{ion}^{thermal} \downarrow \quad \Rightarrow \quad \text{radiation}$$

(
- $E_{e-gas}^{thermal}$ cannot be deliberated because of the filled Fermi-sea
- neutrino emission is only important very early when $T_* > 10^8\ K$

)

Specific heat capacity per ion: $c_v = \frac{3}{2}k \quad \text{(erg/K)} \ \text{for a monatomic gas} \ \text{(e.g. C}^+\text{-ions)}$

Total thermal energy: $U = c_v \cdot T \cdot N_{ions} \quad = \frac{3}{2}kT\frac{M}{Am_u} \qquad \left(\approx 10^{48}\ erg,\ T_* = 10^7\ K\right)$

$$L = -\frac{dU}{dt} \quad \Leftrightarrow \quad C \cdot M \cdot T^{3.5} = \frac{d}{dt}\left(\frac{3}{2}kT\frac{M}{Am_u}\right)$$

Integrate…

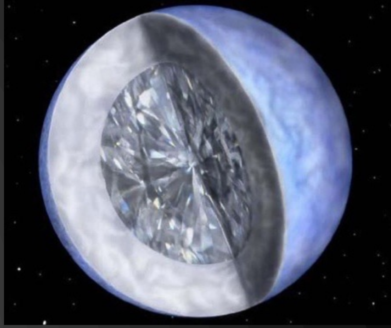
$(T_0 \gg T)$ $\Rightarrow$ $\boxed{\tau \propto \left(\frac{M}{L}\right)^{5/7}}$ **cooling age!**

Ex.23

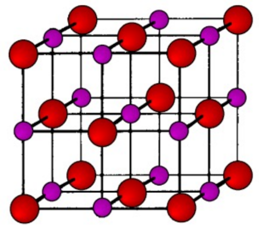
Answer: Crystallization!

Problem: estimated WD cooling ages were too large by a factor 10 $\tau_{WD} \gg \tau_{cluster}$

Cooling of White Dwarfs



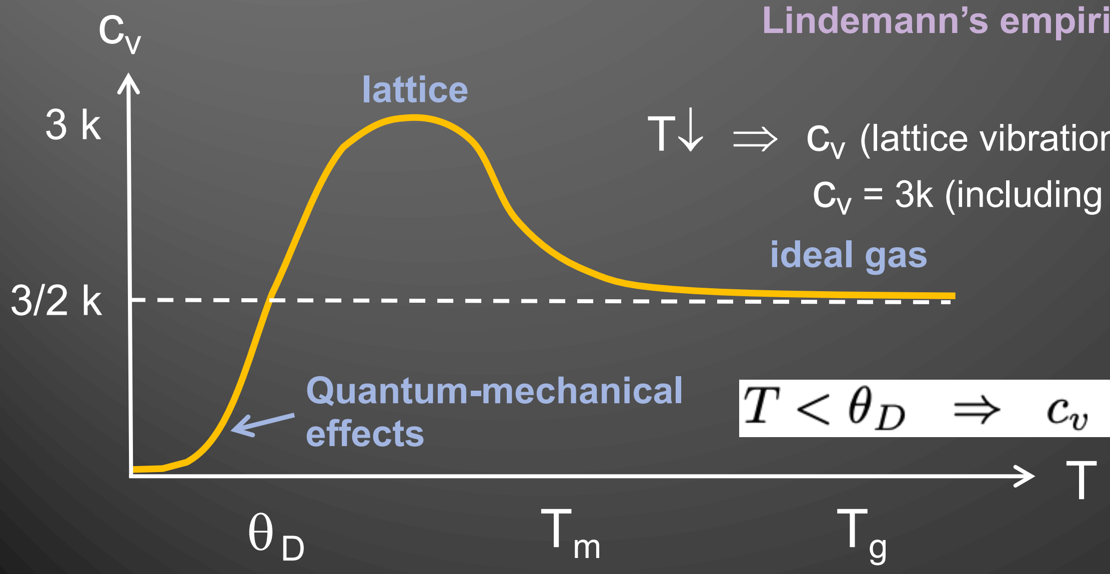
Crystallization of ion lattice



Formation of lattice:

$$\frac{\langle \delta(r_i)^2 \rangle}{r_i^2} < \frac{1}{16} \qquad \leftrightarrow \qquad \frac{E_{coulomb}}{E_{thermal}} = \frac{Z^2 e^2 / r_i}{kT} > 171$$

Lindemann's empirical rule



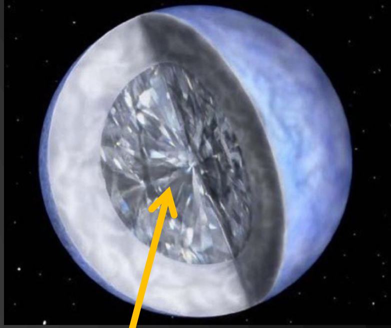
T↓ ⇒ c_V (lattice vibrations) dominate over c_V (free thermal motion)

c_V = 3k (including $E_{pot,lattice}$ → ½ k per mode)

$$T < \theta_D \;\Rightarrow\; c_v \downarrow\downarrow \;\Rightarrow\; T \downarrow\downarrow \quad \text{rapid cooling}$$

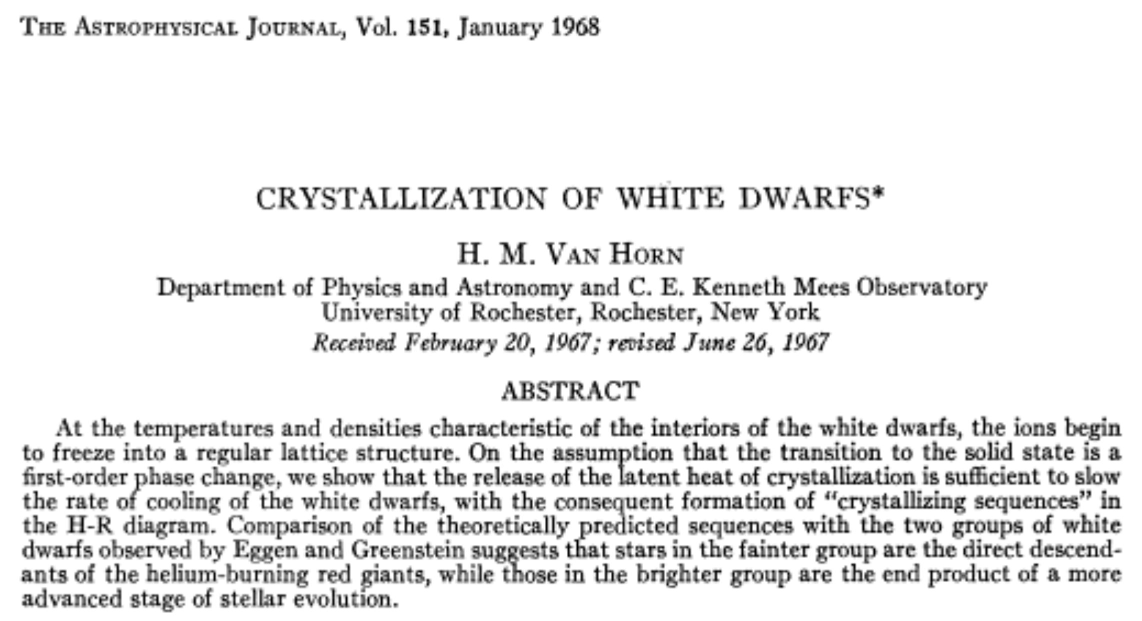
observational consequences

Cooling of White Dwarfs



diamond WD!

”Lucy” *



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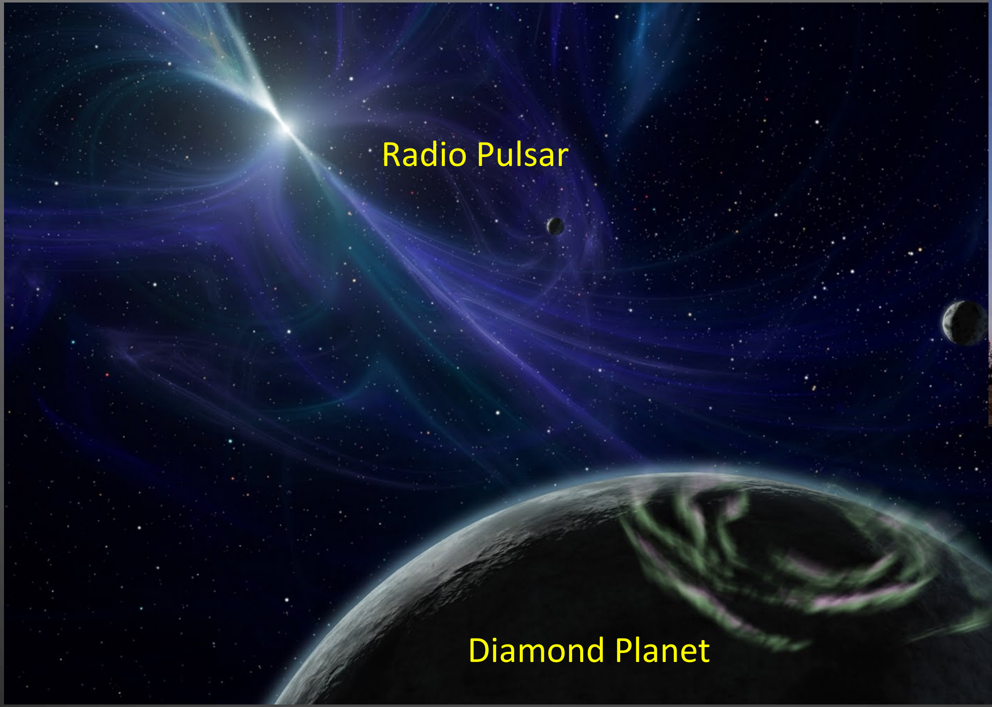
* Beatles song: ”*Lucy in the Sky with Diamonds*”

Cooling of White Dwarfs

Bailes et al. (2011), Science



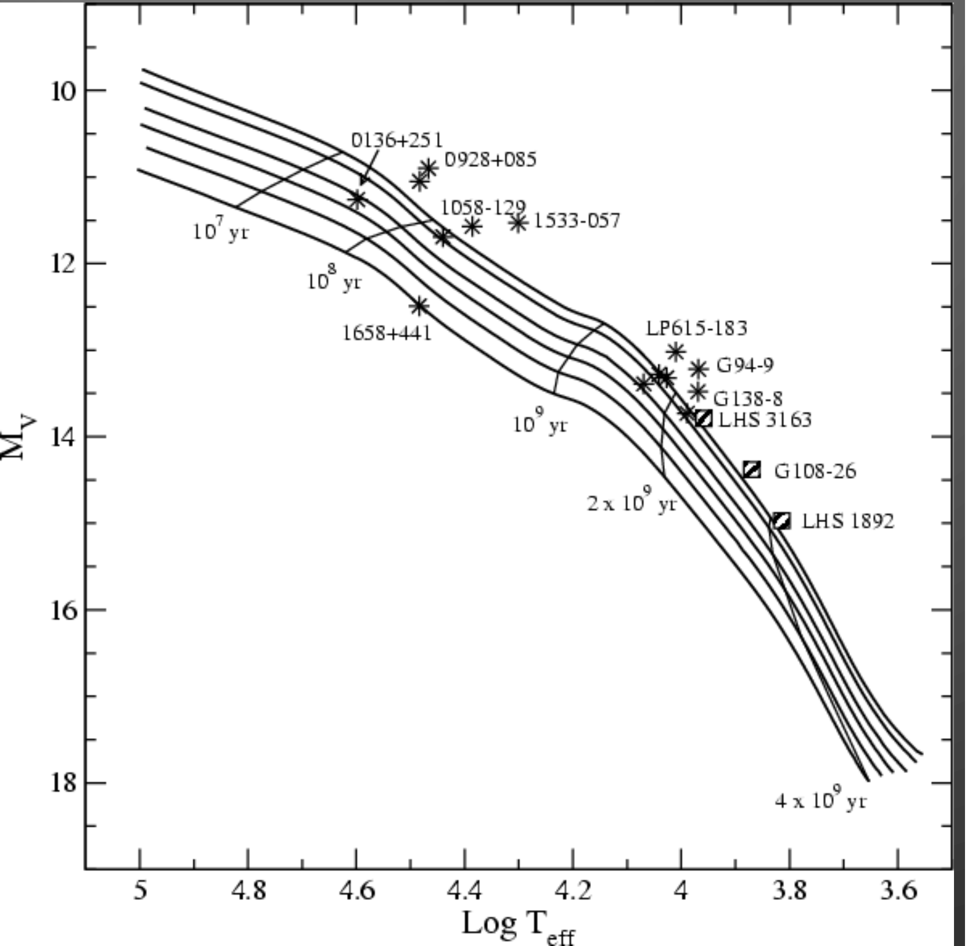
Parkes Radio Telescope

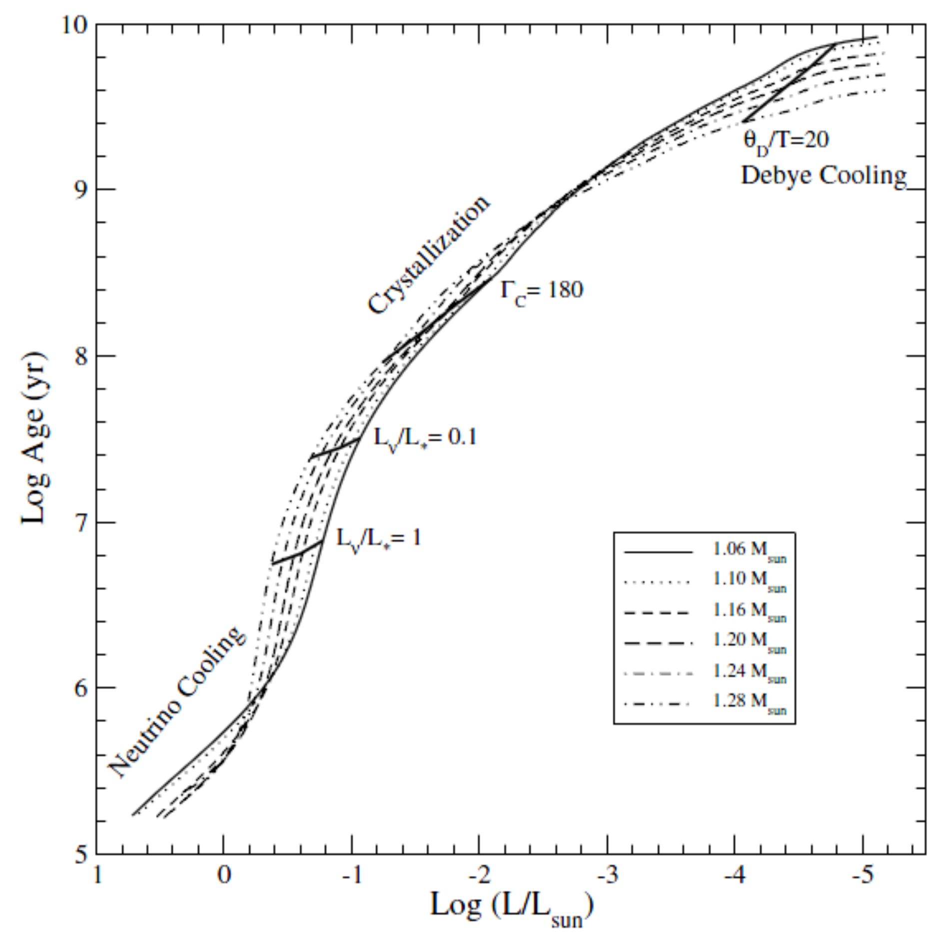
Observations of cooling WDs

testing cooling models

Althaus et al. (2007), A&A 465, 249
Massive 1.06-1.28 $M_\odot$ ONeMg WDs with
helium/hydrogen envelopes

see also
Ex.24





Observations of cooling WDs

testing cooling models

Hansen et al. (2007), ApJ 671, 380
The WD cooling sequence of NGC 639

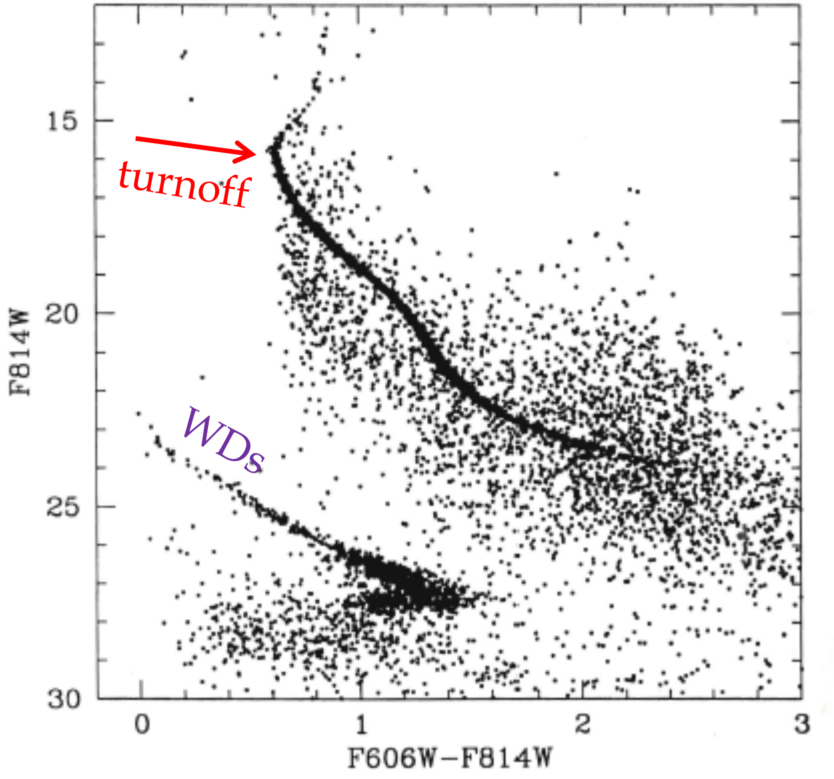
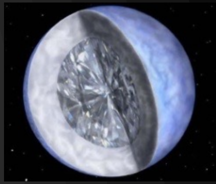
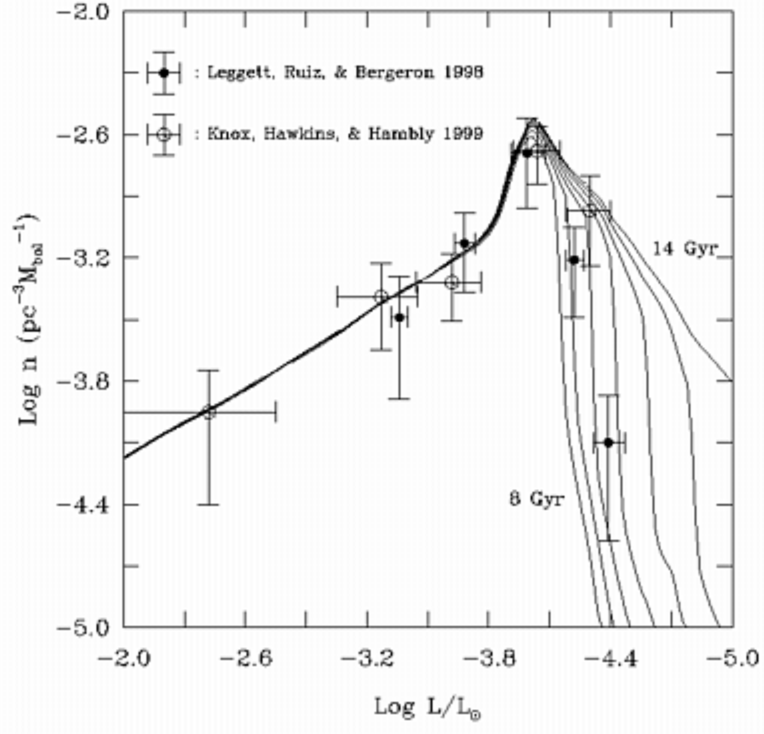


FIG. 1.—ACS color-magnitude diagram for our field in NGC 6397. All real point sources are shown (so the extended galaxy population is not shown). Prominent features include a cluster main sequence, a clear main-sequence turnoff, and a clear white dwarf cooling sequence. Most important is the clear evidence for a sharp decline in the number of white dwarfs at magnitudes greater than F814W = 27.6. The detectability of sources at fainter magnitudes is evident from the fainter, bluer population of background galaxies that survive the point source cuts.

Determine ages of clusters or Gal. disk



Gilles Fontaine (2000)
Evidence for rapid cooling…



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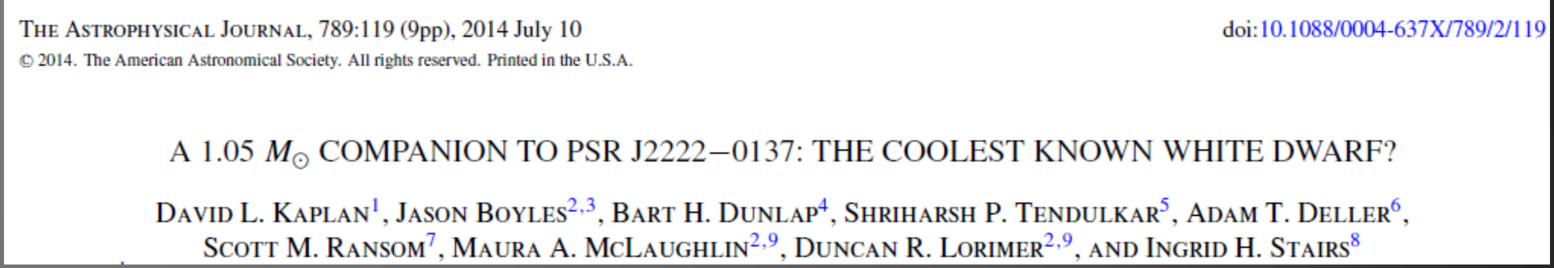

The Astrophysical Journal, 789:119 (9pp), 2014 July 10 doi:10.1088/0004-637X/789/2/119

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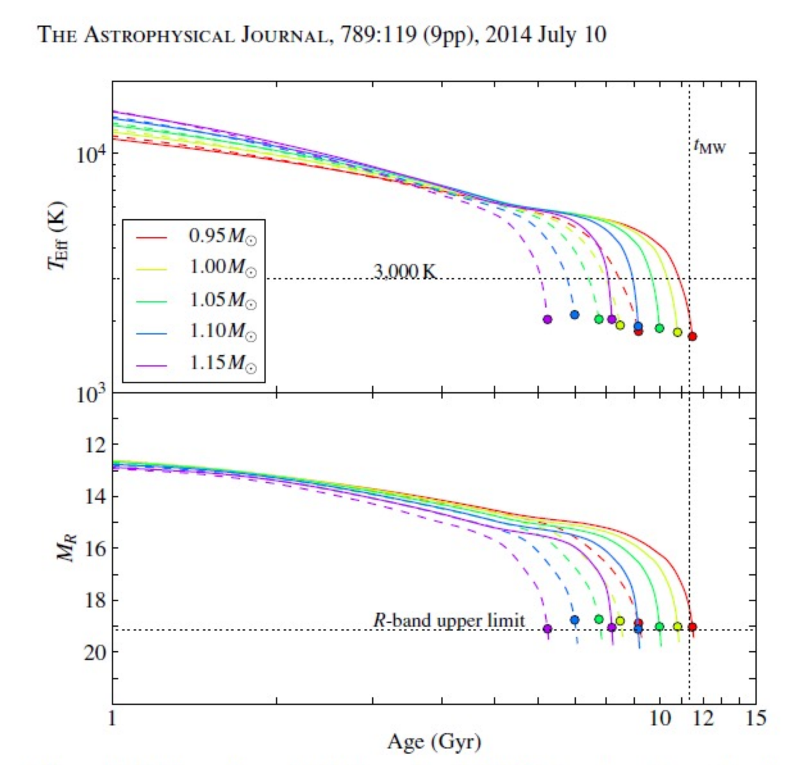
A 1.05 $M_\odot$ COMPANION TO PSR J2222−0137: THE COOLEST KNOWN WHITE DWARF?

David L. Kaplan[1], Jason Boyles[2,3], Bart H. Dunlap[4], Shriharsh P. Tendulkar[5], Adam T. Deller[6], Scott M. Ransom[7], Maura A. McLaughlin[2,9], Duncan R. Lorimer[2,9], and Ingrid H. Stairs[8]



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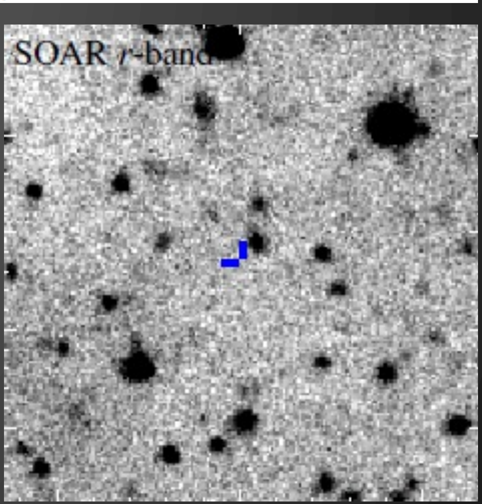
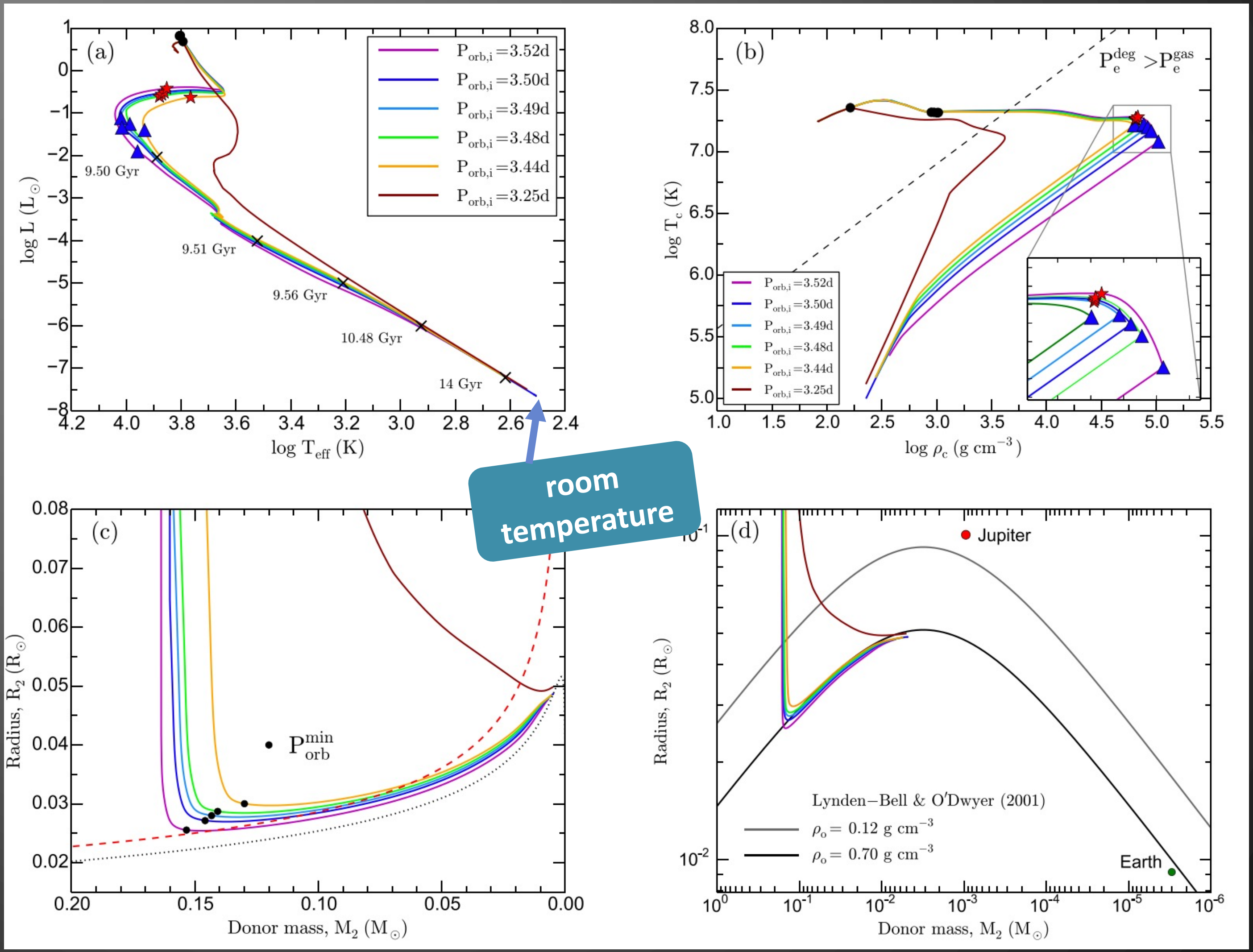


Theoretical Models of cooling WDs

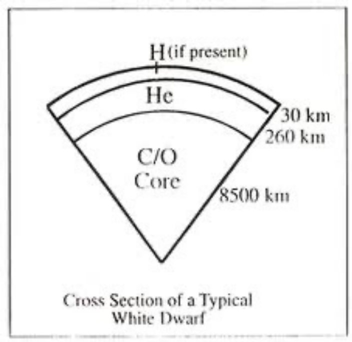


room temperature

Sengar, Tauris, Langer, Istrate (2017), MNRAS Letters

Cooling of White Dwarfs

Summary



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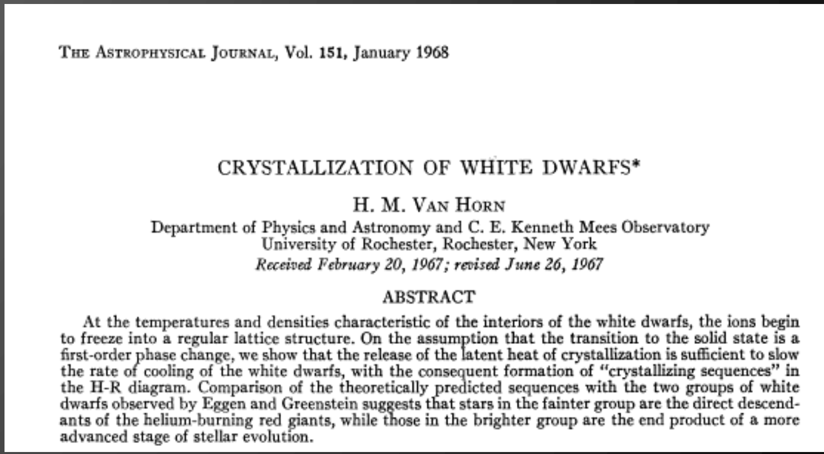

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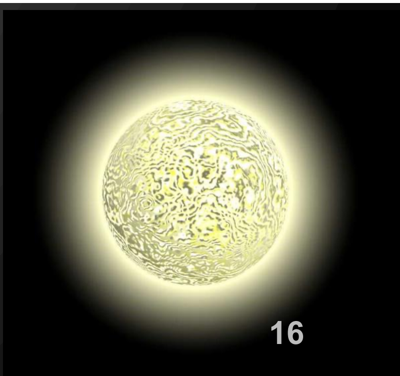
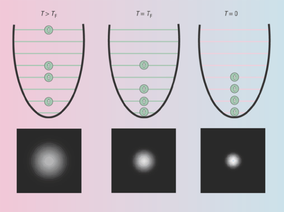
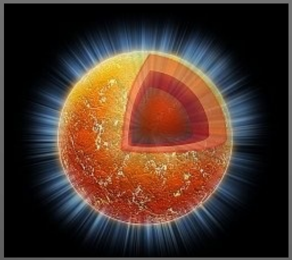
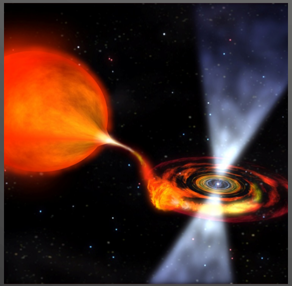
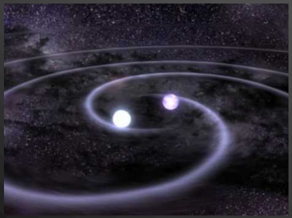


Programme









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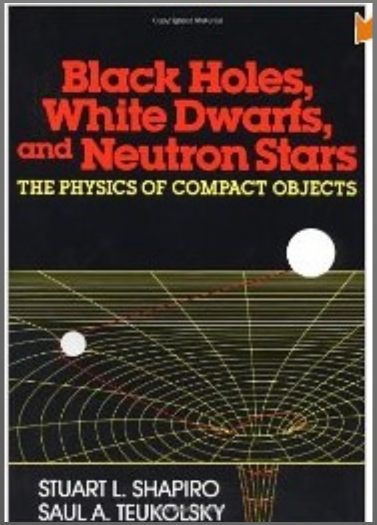
Physics of Compact Objects
week 4



Black Holes, White Dwarfs, and Neutron Stars
THE PHYSICS OF COMPACT OBJECTS
STUART L. SHAPIRO
SAUL A. TEUKOLSKY

Shapiro & Teukolsky (1983), Wiley-Interscience

Curriculum
- Chapter 4: p.82-87, (91-92), 100-105

Exercises: #23, 24
- Mon. Sep.25, 10:15-12:00

Next lecture: NS structure and EoS above neutron drip
S&T Chapters 8+9.
- Mon. Oct.2, 08:15-10:00, Aud. 2.115