AND THEIR BINARY INTERACTIONS



AALBORG UNIVERSITY

Thomas Tauris – Physics, Aalborg University

Programme











* Introduction

* Degenerate Fermi Gases

Non-relativistic and extreme relativistic electron / (n,p,e⁻) gases

* White Dwarfs

Structure, cooling models, observations

* Neutron Stars

Structure and equation-of-state

* Radio Pulsars

Characteristics, spin evolution, magnetars, observations, timing

* Binary Evolution and Interactions

X-ray binaries, accretion, formation of millisecond pulsars, recycling

* Black Holes

Observations, characteristics and spins

- * Gravitational Waves
 Sources and detection, kilonovae
 - * Exam

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Last week Cooling of White Dwarfs

- Surface layers and core structure
- Photon diffusion equation $L = -4\pi r^2 \frac{c}{3\kappa\rho} \frac{d}{dr} (aT^4)$
 - Temperature gradient
 - Pressure gradient (via hydrostatic equilibrium)
 - Core-surface boundary conditions
 - Luminosity as a function of (M, T_{*})
- Elementary treatment of WD cooling
 - Residual ion thermal energy
 - Cooling age
- Crystallization
 - Rapid cooling
- Observational support of WD cooling models

Vol. 151, January 1968

Blackboard

CRYSTALLIZATION OF WHITE DWARFS*

H. M. VAN HORN Department of Physics and Astronomy and C. E. Kenneth Mees Observatory University of Rochester, Rochester, New York Received February 20, 1967; revised June 26, 1967

ABSTRACT

At the temperatures and densities characteristic of the interiors of the white dwarfs, the ions begin to freeze into a regular lattice structure. On the assumption that the transition to the solid state is a first-order phase change, we show that the release of the latent heat of crystallization is sufficient to slow the rate of cooling of the white dwarfs, with the consequent formation of "crystallizing sequences" in the H-R diagram. Comparison of the theoretically predicted sequences with the two groups of white dwarfs observed by Eggen and Greenstein suggests that stars in the fainter group are the direct descendants of the helium-burning red giants, while those in the brighter group are the end product of a more advanced stage of stellar evolution.



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Cold equations-of-state above neutron drip

• EoS for $\rho_{drip} < \rho < \rho_{nuc}$

- Baym-Bethe-Pethick (BBP) EoS
- Stability of NSs
- EoS for $\rho > \rho_{nuc}$
 - Nucleon-nucleon interactions
 - Muons, hyperons, Δ -resonances, pion/kaon condensation
 - Superfluidity (glitches/cooling of NSs)
 - Bethe-Johnson (BJ) EoS
 - Quark (strange) stars / quark-novae
- Summary of EoS above neutron drip
- Structure of NSs
 - Cross section
 - Soft vs Stiff EoS
 - Observational constraints on M and R



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EoS above neutron drip

■ $\rho_{drip} \approx 4 \times 10^{11} \text{ g cm}^{-3}$ Neutron drip (nuclei, e⁻, n) 2 phase system ■ $\rho_{drip} < \rho < \rho_{nuc} = 2.8 \times 10^{14} \text{ g cm}^{-3}$ Fairly well-known EoS (e.g. BBP)

- $\rho > \rho_{nuc}$ not well understood. Problems:
 - nucleon-nucleon interactions
 - many-body problem
 - hyperons (nucleon-like strange baryons)
 - pion/kaon condensation

 $\rho > 10 \rho_{nuc}$ ultra-high densities:

- no relativistic many-body Schrödinger equation is known
- "meson clouds" around nucleons quark-drip (break-down of potential, no longer 2-body interactions)
- neutron lattice?



repulsion









Nucleon-nucleon interactions

The exchange of vector mesons (S=1) induces **repulsive** NN forces, while the exchange of scalar mesons (S=0) induces **attractive** forces.

The two lowest mass vector mesons are: ρ (769 MeV), ω (783 MeV).

The intermediate-range attractive NN force is caused by the σ (f₀) meson (600 MeV), and the long-range NN force by π (140 MeV).

The Yukawa-like potential: V_{12} =

$$=\pm g^2 \frac{e^{-\mu r}}{r}$$

approximately describes the NN interactions.

$$E_V = \frac{1}{2} \sum_{i \neq j} V_{ij}$$

(sum all pairs of NN interactions)

$$1/\mu = 1/\alpha m =$$
approx. range
 $\alpha =$ scaling constants, $m =$ mass



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g, a

Baym-Bethe-Pethick (BBP) EoS

- "Compressible liquid drop model"
- Reid soft core: superposition of Yukawa-like potentials $V_{12} = \pm g^2 \frac{e^{-\mu}}{2}$

Minimizing the total energy density: $\varepsilon = n_N \cdot (W_N + W_L) + \varepsilon_n (1 - V_N n_N) + \varepsilon'_e(n_e)$ for constant *n* with respect to *A* fraction of volume which is gas

 $n = A \cdot n_N + (1 - V_N n_N) \cdot n_n$ (baryon number density)

Nuclei must be stable against
$$\beta$$
-decay (Z ~ const
 $n \leftrightarrow p + e^- + \overline{\nu}_e$ $\mu_n^N = \mu_p^N + \mu_e$ ($\mu_\nu = 0$, ν leaves the star)

- Free n-gas must be in equilibrium with neutrons inside nuclei: $\mu_n^G = \mu_n^N$
- Pressure balance between n-gas and nuclei: $P_n^G = P_n^N$

iS



Baym-Bethe-Pethick (BBP) EoS

Fig. 8.1 + 8.2 (Bayn, Bethe & Pethick 1971).





EoS above nuclear density



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EoS above nuclear density

Fig. 9.2 + 9.3 (Baym & Pethick 1979).



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A "**soft**" equation of state has an average system energy which is attractive at nuclear densities. (e.g. a Reid potential).

A "stiff" equation of state has a repulsive component at higher densities.



soft EoS: *P* is small (Γ is small) $\rightarrow R$ is small, ρ_c is large (M_{max} is small) stiff EoS: *P* is large (Γ is large) $\rightarrow R$ is large, ρ_c is small (M_{max} is large) $P=K\rho^{\Gamma}$



Stability of Compact Objects



Bethe-Johnson (BJ) EoS

- $\rho_{nuc} < \rho < 3 \times 10^{16} \ g \ cm^{-3} \qquad (n, p, \Lambda, \Sigma^{\pm, 0}, \Delta^{\pm, 0, ++})$
- Modified Reid soft core (includes the repulsive terms at small distances)
- Variational calculation (mathematical analysis to find minimum ε)
- Hartree-Fock analysis (many-body interactions)
- Stiff EoS: $P \propto \rho^{2.54}$



EoS above nuclear density

Steiner, Lattimer & Brown (2012)

Lattimer (2007)



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Adding exotic particles to the NS structure

Muons

Muon contribution to EoS: $\mu_e > m_\mu c^2$ (106 MeV, $\rho \approx 2.4 \times 10^{14} \ g \ cm^{-3}$) $e^- \leftrightarrow \mu^- + \overline{\nu}_{\mu} + \nu_e$ equilibrium: $\mu_{\mu} = \mu_e$, $\mu_n = \mu_p + \mu_e$ Charge neutrality: $n_p = n_e + n_\mu$ ν_{μ} $\overline{\nu}_e$ Aalborg, Autumn 2023 **Thomas Tauris** 22



Hyperonic soup

Fig.6.16 (Camenzind 2007)



Fig. 6.16. Population of various particles in hadronic stars as a function of baryon density. Nuclear density corresponds to 0.153 fm^{-3} . Figure adapted from Glendenning [5]

Hyperonic soup

Camenzind (2007)

Table 6.2. Baryon octet and	l mesonic states relevant	for neutron star matter
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Particle	m (MeV)	Spin	Isospin	B	Q	S
N	939	1/2	1/2	1	0,1	0
Λ	1115	1/2	0	1	0	-1
$\boldsymbol{\Sigma}$	1190	1/2	1	1	-1,0,1	-1
Ξ	1315	1/2	1/2	1	-1,0	-2
σ	800	0	0	0	0	0
ω	782	1	0	0	0	0
Q	770	1	1	0	-1,0,1	0
π	139	0	1	0	-1,0,1	0
K^+	494	0	1/2	0	1	1
K^0	494	0	1/2	0	0	1
$ar{K}^0$	494	0	1/2	0	0	-1
K ⁻	494	0	1/2	0	-1	-1



Pion condensation

$n \rightarrow p + \pi^ (\mu_n - \mu_p = \mu_e > 140 \text{ MeV} \quad \rho > \rho_{nuc})$

- π have spin S=0 (bosons) and can form a Bose-Einstein condensate.
- Thus in their lowest energy state (z=0) they have no momentum and therefore they do not contribute to the pressure P.
- Pion condensates therefore results in soft EoS
- Kaon condensates may form too



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Superfluidity

- A fermionic superfluid may form at low temperatures.
- Zero-viscosity due to Cooper pairs (BCS theory).
- Three types:
 - $^{1}S_{0}$ neutron superfluid inner crust
 - ${}^{3}P_{2}$ neutron superfluid core
 - $^{1}S_{0}$ proton superfluid core
- Consequences:
 - 1) Formation of vortices
 - 2) Dynamical evolution: pulsar glitches
 - 3) The cooling of NSs
 - 4) The Meissner effect (B-flux tubes)



Superfluidity – Pulsar Glitches



 $I \equiv I_c + I_n$ Moment of inertia

Weak coupling between c: crust + charged particles n: superfluid neutrons in core (Two component model).

A glitch is quickly (minutes) communicated to the charged particles via the B-field, but very slowly (months) to the superfluid neutrons.



The relaxation depends on the pinning/unpinning between **core superfluid vortices** and the normal component of **nuclei (lattices) in the inner crust**, transferring angular momentum.

Problem: the two-component is too simplified and does not explain data (healing parameter Q and relaxation time τ differ for different glitches from the same pulsar)

starquake

Superfluidity – Neutron Star Cooling



Superfluidity – Neutron Star Cooling

Superfluidity affects the neutrino emission processes and the heat capacity.



Note on WD cooling

WD neutrino cooling via the so-called convective URCA process

Inside ions:

 $\mu_e(E_F) < Q: \quad n \to p + e^- + \overline{v}_e \qquad \beta \text{-decay}$ $\mu_e(E_F) > Q: \quad p + e^- \to n + v_e \qquad \text{electon capture}$

Idea of **convective URCA process:** URCA matter is a working fluid for a heat engine driven by convection through the URCA shell





Summary: EoS above nuclear density

Table 8.2 S&T

Table 8.2 Representative Equations of State Above Neutron Drip						
Equation of State	Density Range (g cm ⁻³)	Composition	Interactions	Many-Body Theory		
Ideal neutron gas (Oppenheimer and Volkoff, 1939; OV)	$0 \le \rho \le \infty$	n	None	Noninteracting neutrons.		
Baym-Bethe-Pethick (1971a; BBP)	$4.3 \times 10^{14} < \rho \le 5 \times 10^{14}$	e , n, and equilibrium nuclide	Reid soft core	Mass formula for nuclei constructed from compressible liquid-drop model		
Reid (Pandharipande, 1971; R)	$\rho > 7 \times 10^{14}$	n	Reid soft core adapted to nuclear matter	Variational principle applied to correlation function		
Bethe-Johnson (1974; BJ)	$1.7 \times 10^{14} \le \rho \le 3.2 \times 10^{16}$	$\begin{array}{c} n, p, \Lambda, \Sigma^{+,0} \\ \Lambda^{\pm,0}, \text{ and } \Lambda \end{array}$	Modified Reid : soft core	Constrained variational method		
Tensor-Interaction (Pandharipande and Smith, 1975a; TI)	$\rho > 8.4 \times 10^{13}$	n	Nuclear attraction due to pion exchange tensor interactions	Constrained variational method		
Three-nucleon interaction (Friedman and Pandharipan 1981; TNI)	$\rho > 1.7 \times 10^{14}$ ide	a	Two- and three- nucleon interactions	Constrained variational method		
Mean field '(Pandharipande and Smith, 1975b; MP)	ρ > 4.4 × 10 ¹¹	8	Nuclear attraction due to scalar exchange	Mean field approxi- mation for scalar; variational method		
Relativistic mean field (Walecka, 1974; RMF)	$\rho > 1.7 \times 10^{14}$	n	Relativistic mean field scalar plus vector exchange fitted to nuclear matter	Relativistic mean field approximation		

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PULSARS: SPINNING NEUTRON STARS WITH HIGH B-FIELDS





NS radii can be determined for a few young NSs (the magificent seven) by fitting blackbody spectra:

$$L = 4\pi R^2 \sigma T^4 \qquad \wedge \qquad F = \frac{L}{4\pi d^2}$$
$$R = d \sqrt{\frac{F}{\sigma T^4}}$$

Correction for the gravitational redshift:

$$R_{\infty} \approx \frac{R}{\sqrt{1 - 2\frac{GM}{c^2 R}}} \qquad \wedge \qquad T_{\infty} \approx T\sqrt{1 - 2\frac{GM}{c^2 R}}$$

The apparent (observed) radius is larger than the true radius

In practice more difficult because of the unknown spectral hardening (atmospheric corrections), and uncertainties in distance estimates.







Figure 1. Blackbody fits to the optical and X-ray spectra of RX J1856.5-3754 for a two-component model (a) and a model with a continuous temperature distribution (b).

Watts A L, Yu W, Poutanen J, Zhang S, et al Sci. China-Phys. Mech. Astron. January (2017) Vol. 60 No. 1





FIGURE 1. The relationship between the composition and inter-particle forces in the neutron star core, the EOS, the mass-radius relation, and the exterior space-time of the star. The space-time of the rotating neutron star imprints its signature on radiation emitted from the stellar surface: we can use this to infer the EOS.

Watts (2019)

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eXTP mission (launch 2025), joint Chinese + EU mission

Radiation Radius: Nearby Neutron Star

Lattimer (2009)



Antoniadis et al. (2013)



- Shapiro delay (range and shape) measurements of binary radio pulsars.
- Measurements of other post-Keplerian parameters: $\dot{\omega}$, \dot{P}_b , γ
- Dual-line spectroscopy (measurements of WD spectra yields a (dopplershift, besides from radio pulsar timing).





Nice (2013)



Any PK measurement yields a line in the (m_1, m_2) -plane. Hence, two PK parametres determines m_1 and m_2 uniquely.

• The double pulsar PSR J0737-3039





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Kramer et al. (2021)



PSR J0348+0432 M=2.01 ± 0.04 Antoniadis et al. (2013)

www.stellarcollapse.org/nsmasses

Lattimer (2013)





Bimodal NS mass distribution (reflecting bimodal NS birth mass distribution)

Antoniadis et al. (2016)

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Neutron star moment of inertia



Kehl et al. (2016)

Measurement of the Lense-Thirring effect in the double pulsar system will allow to constrain the NS moment of inertia:

$$I = k^2 M R^2$$

Fig.6.7 (Camenzind 2007)





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See Weber (2005) Prog.Part.Nucl.Phys.54:193-288, for a review.

- MIT bagmodel
 - degenerate Fermi sea of massless quarkes
 - energy density B
 - important physics parameters: B, m_s , α_s (bag constant, mass of strange quark, strong interaction coupling constant)

• EoS:
$$P = \frac{1}{3}(\rho - 4B)$$

- M(R) (quark stars with larger masses have larger radii)
- Difficult to confirm observationally (sub-ms pulsar: P < 0.6 ms)
- Hybrid stars are very popular: quark core + normal matter
- Quark-novae represent the transition from a normal NS to hybrid star







Hybrid Star



Figure 1: Competing structures and novel phases of subatomic matter predicted by theory to make their appearance in the cores ($R \lesssim 8$ km) of neutron stars [2].

Quark-novae



A quark-nova is the violent explosion resulting from the conversion of a neutron star to a quark star (Oyued, Dey & Dey, A&A 390 L39-42, 2002).

When a neutron star spins down, it may convert to a quark star through a process known as quark deconfinement.

Direct evidence for quark-novae is lacking; however, observations of supernovae SN 2006gy, SN 2005gj and SN 2005ap have been suggested may point to their existence (Leahy & Ouyed, MNRAS 387, 1193, 2008).



Neutron star / black hole mergers and constraints on the NS EoS



Tidal deformation and NS EoS



Constraining the NS EoS from tidal interactions in GW170817

Class. Quantum Grav. 37 (2020) 045006

B P Abbott et al



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Kilonovae





Metzger & Berger(2012)

Excellent reviews: Giacomazzo, Eichler & Arcones (2019) Shibata & Hotokezaka (2019)

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The merger remnant also places a constraint on the maximum neutron star mass

The remnant NS cannot be long lived, or there would be too much energy in the EM observantion

 $M_{\rm max} \le 2.17 M_{\odot} \ (90\%)$

Margalit and Metzger ApJL 850 19 (2018)



Coughlin, Dietrich, Margalit, Metzger arXiv:1812:04803

nature astronomy

Relativistic Shapiro delay measurements of an extremely massive millisecond pulsar

H. T. Cromartie^{®1*}, E. Fonseca^{®2}, S. M. Ransom^{®3}, P. B. Demorest⁴, Z. Arzoumanian⁵, H. Blumer^{6,7}, P. R. Brook^{6,7}, M. E. DeCesar⁸, T. Dolch⁹, J. A. Ellis¹⁰, R. D. Ferdman^{®11}, E. C. Ferrara N. Garver-Daniels^{6,7}, P. A. Gentile^{6,7}, M. L. Jones^{6,7}, M. T. Lam^{6,7}, D. R. Lorimer^{6,7}, R. S. Lynch¹⁴, M. A. McLaughlin^{6,7}, C. Ng^{15,16}, D. J. Nice^{®8}, T. T. Pennucci^{®17}, R. Spiewak^{®18}, I. H. Stairs¹⁵, K. St J. K. Swiggum¹⁹ and W. W. Zhu²⁰

Despite its importance to our understanding of physics at supranuclear densities, the equation of state (EoS) of matter deep within neutron stars remains poorly understood. Millisecond pulsars (MSPs) are among the most useful astroPrecise neutron star mass measurements are an effect constrain the EoS of the ultradense matter in neutron sta Although radio pulsar timing cannot directly determin star radii, the existence of pulsars with masses exceeding

we have measured the mass of the MSP J0740+6620 to be $2.14^{+0.10}_{-0.09} M_{\odot}$ (68.3% credibility interval; the 95.4% credibility interval is $2.14^{+0.20}_{-0.18} M_{\odot}$). It is highly likely to be the most massive neutron star yet observed, and serves as a strong constraint on the neutron star interior EoS.







Fig. 1| Timing residuals from all observations of J0740+6620 as

summary Structure of Neutron Stars

Cold equations-of-state above neutron drip

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Physics of Compact Objects week 5



Shapiro & Teukolsky (1983), Wiley-Interscience

Curriculum

- Chapter 8: p.(188-197), (220-240)
- Chapter 9: p.241-253, 253-258.

Want to know more? See our course webpage for modern reviews on the NS EoS: e.g. Lattimer & Prakash (2016), Bauswein et al. (2017).

Next lecture: Chapter 10: p.267–290. + Tauris & van den Heuvel (2023), Chapter 14.1

Exercises: # 5, 6, 12, 14

- Mon. Oct. 2, 10:15-12:00