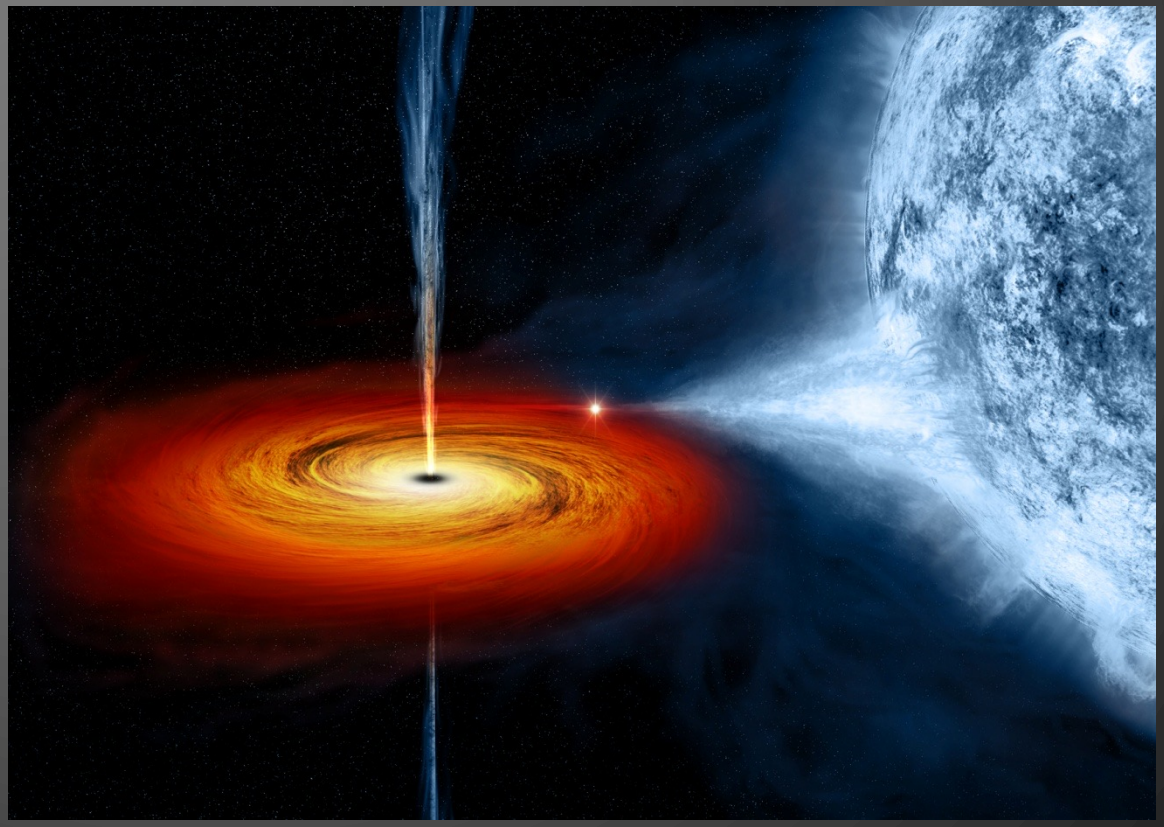
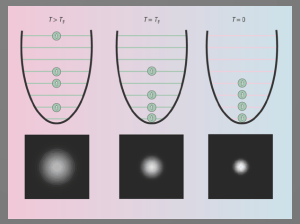
PHYSICS OF COMPACT OBJECTS
AND THEIR BINARY INTERACTIONS

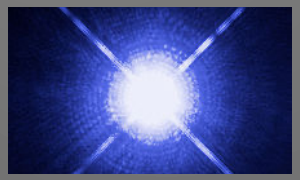


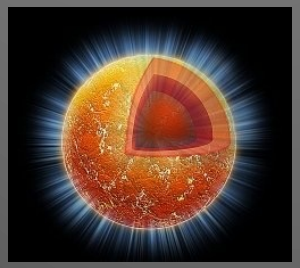
Thomas Tauris – Physics, Aalborg University

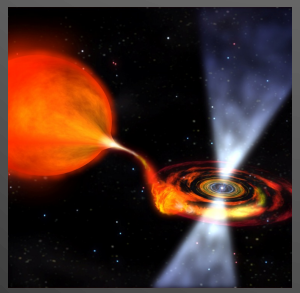
AALBORG UNIVERSITY

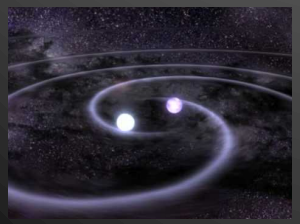
Programme









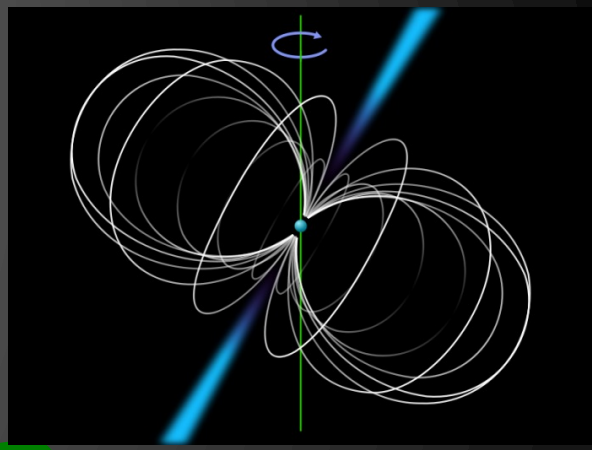


- **Introduction**
- **Degenerate Fermi Gases**
  Non-relativistic and extreme relativistic electron / (n,p,e^-) gases
- **White Dwarfs**
  Structure, cooling models, observations
- **Neutron Stars**
  Structure and equation-of-state
- **Radio Pulsars**
  Characteristics, spin evolution, magnetars, observations
- **Binary Evolution and Interactions**
  X-ray binaries, accretion, formation of millisecond pulsars, recycling
- **Black Holes**
  Observations, characteristics and spins
- **Gravitational Waves**
  Sources and detection, kilonovae
- **Exam**

Last time

Radio Pulsars

Characteristics, observations, spin evolution, magnetars



- Observational aspects of radio pulsars
  - The radio pulsar population in the Milky Way
  - Pulse profiles / Scintillation / Dispersion measure
  - Emission properties
- Spin evolution of pulsars in the $P\dot{P}$-diagram
  - The magnetic dipole model
  - Evolution with B-field decay
  - Evolution with gravitational wave emission
  - The braking index
  - True ages of radio pulsars
- Magnetars
  - Soft gamma-ray repeaters (SGRs) and Anomalous X-ray pulsars (AXPs)

Blackboard

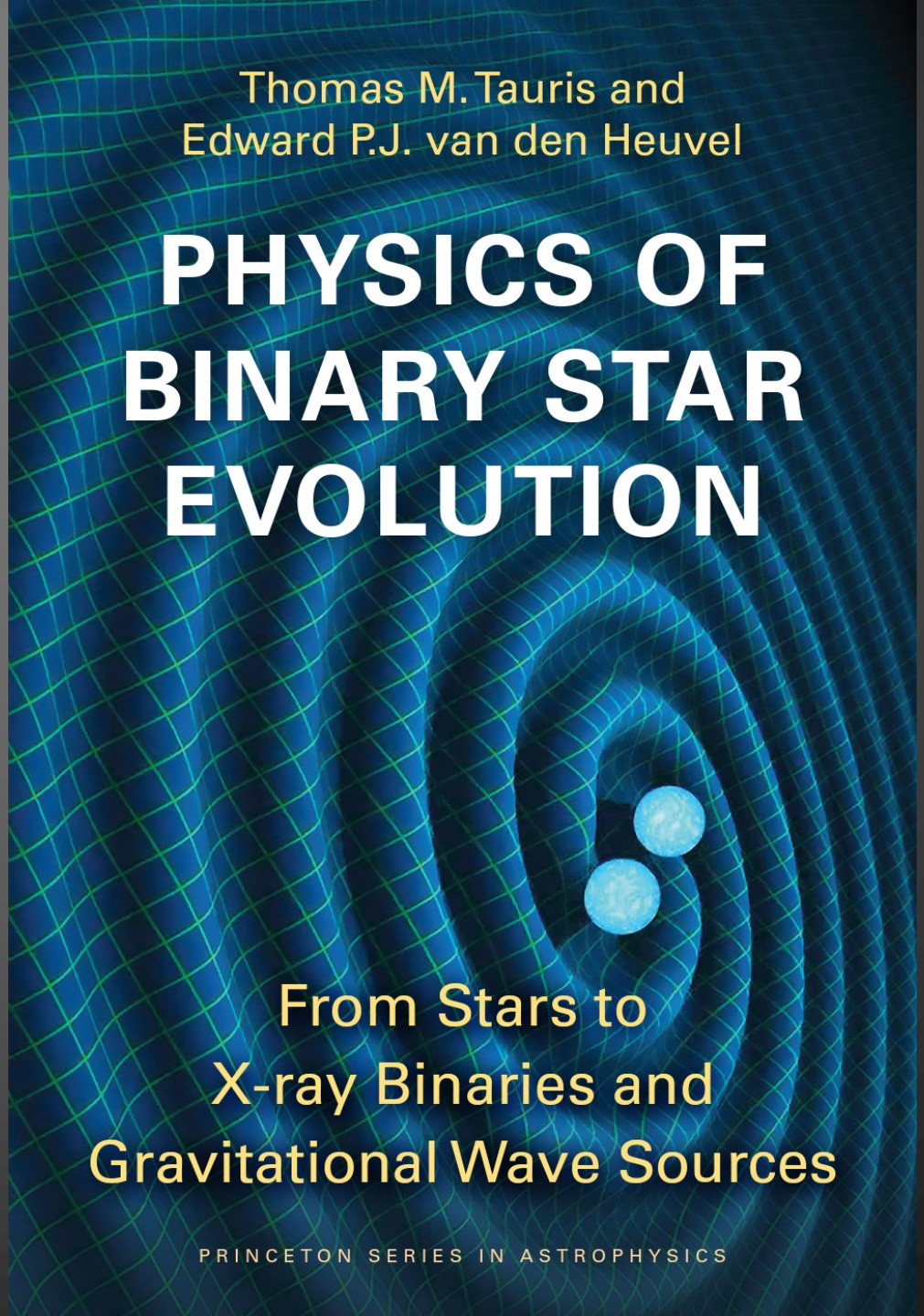
X-ray Binaries

- X-ray binaries (HMXBs / LMXBs)
- Roche-lobe overflow - Cases A, B, C, and Case BB
- Stability criteria for mass transfer / stellar evolution
- Orbital angular momentum balance equation
- Common envelope and spiral-in evolution

For a review: Tauris & van den Heuvel (2006)
and new textbook: Tauris & van den Heuvel (2023)

Thomas M. Tauris and
Edward P.J. van den Heuvel

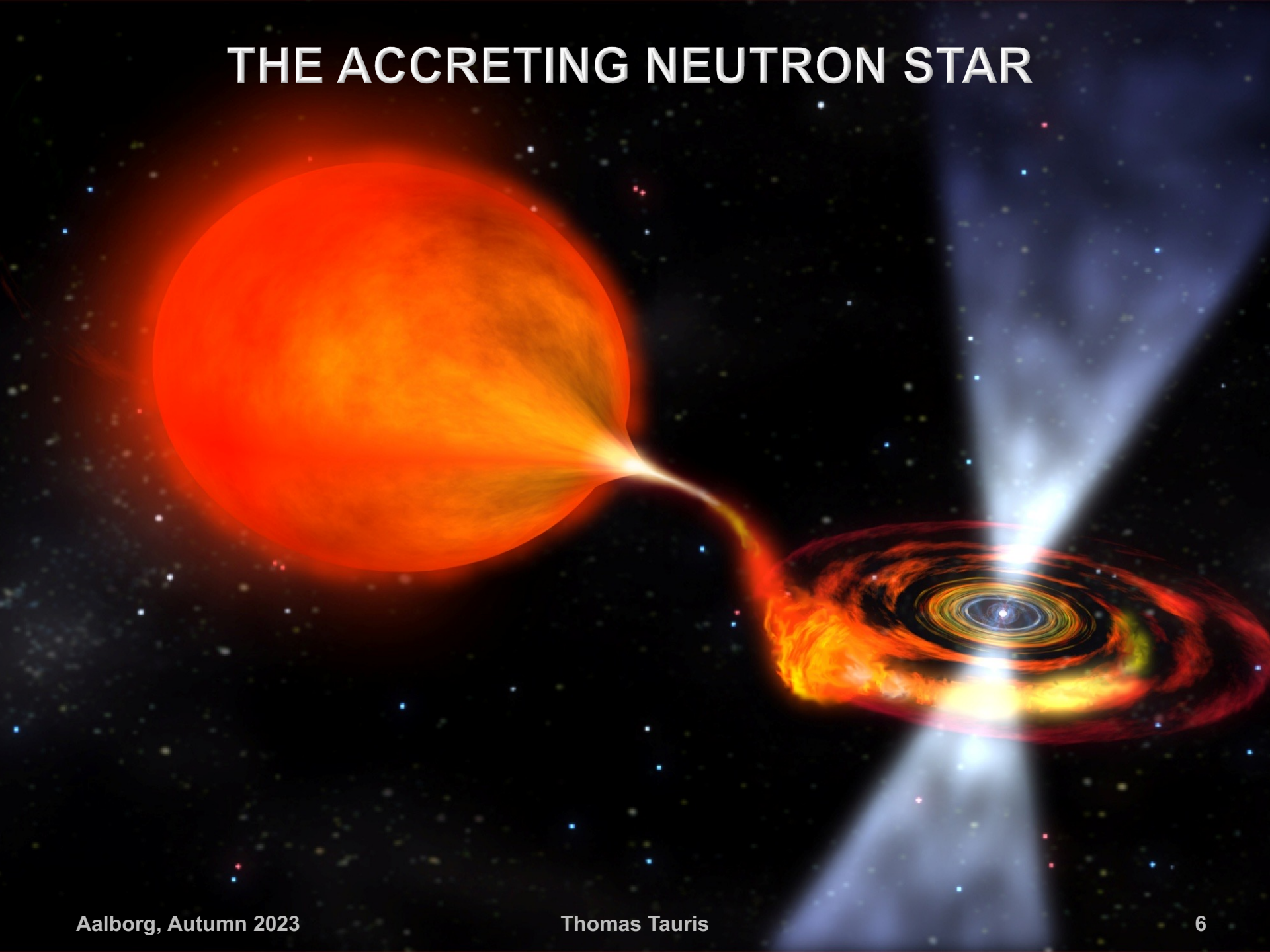
PHYSICS OF BINARY STAR EVOLUTION



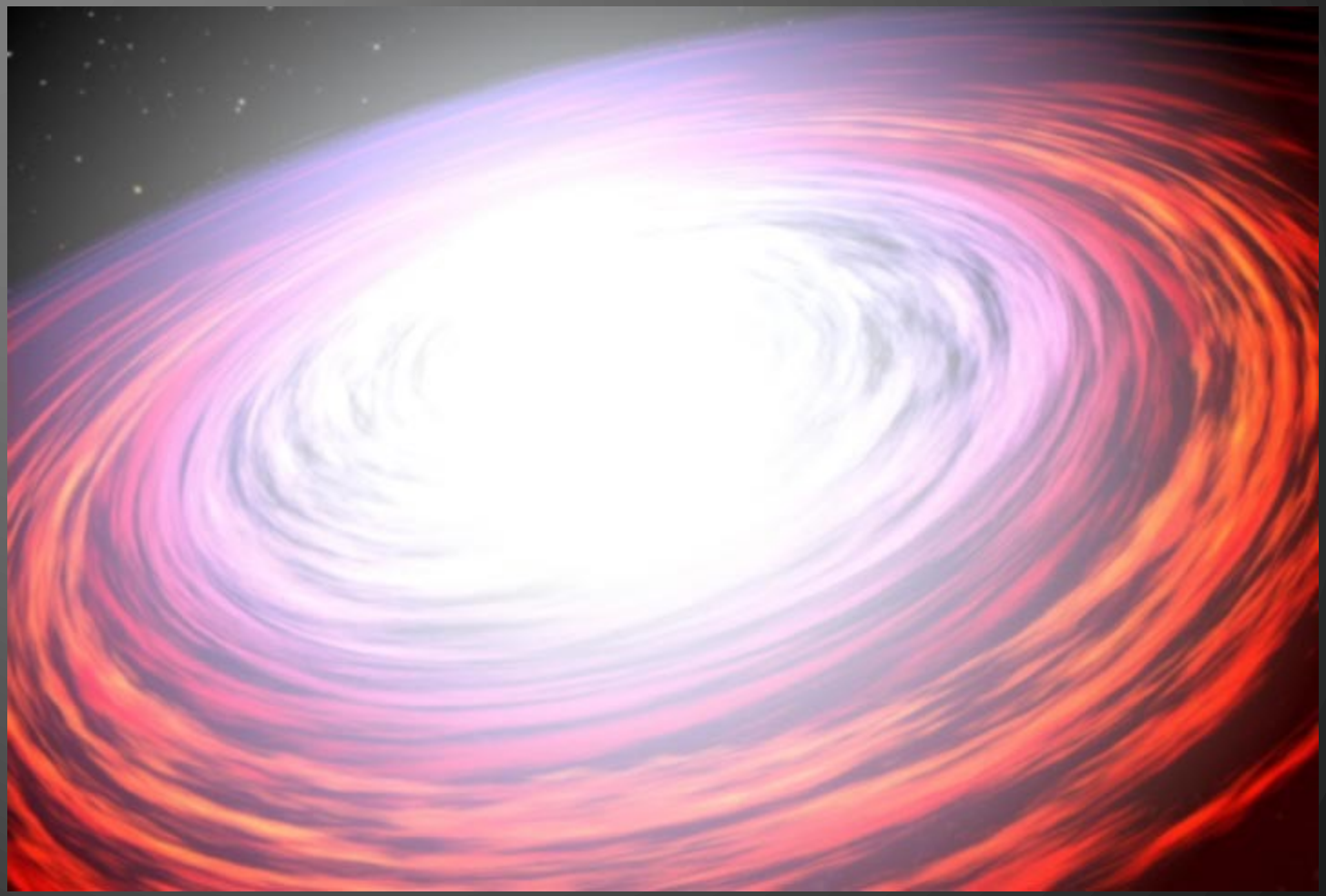
From Stars to X-ray Binaries and Gravitational Wave Sources

PRINCETON SERIES IN ASTROPHYSICS

THE ACCRETING NEUTRON STAR

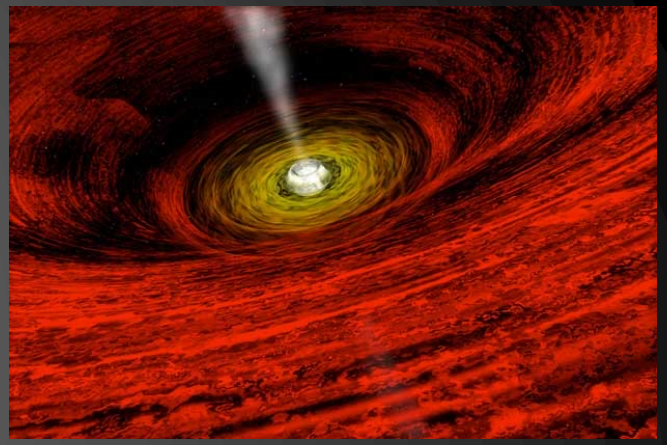


Energetics of accretion



Released gravitational binding energy (infall from ∞)

$$\Delta U = \frac{GMm}{R}$$



$1 M_{\odot}$ accretor and $L_X = 10^{37}\ erg\ s^{-1}$

Stellar object	Radius (km)	$\Delta U/mc^2$	$\Delta U/m$ (erg g^{-1})	dM/dt (M_{sun} yr^{-1})	Column density (g cm^{-2})
Sun	7×10^{5}	2×10^{-6}	2×10^{15}	1×10^{-4}	140
White dwarf	10000	2×10^{-4}	1×10^{17}	1×10^{-6}	16
Neutron star	10	0.15	1×10^{20}	1×10^{-9}	0.5
Black hole	3	0.1-0.4	4×10^{20}	4×10^{-10}	0.3

Note: X-rays are stopped at column densities larger than a few g cm^{-2}

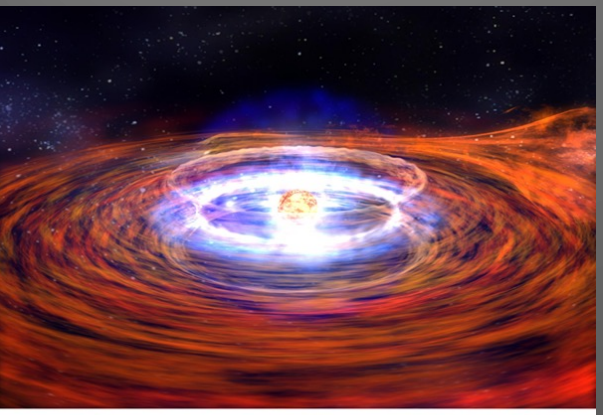
Accretion Luminosity

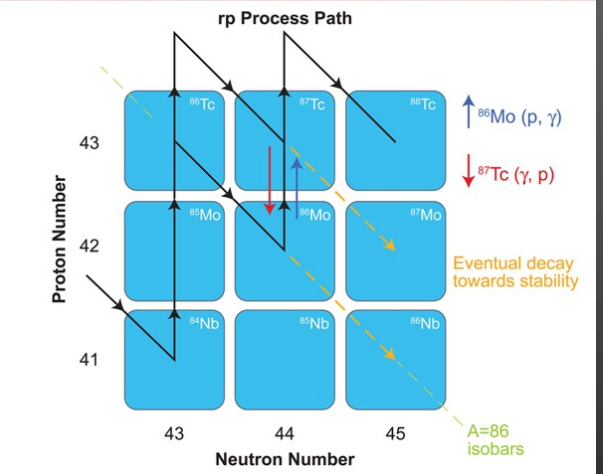
$$L_X = (\varepsilon_{nuc} + \varepsilon_{grav})\,\dot{M}_{acc}$$

Nuclear burning at surface of compact object

$$H \rightarrow He \rightarrow C \rightarrow O \rightarrow Si \rightarrow Fe$$

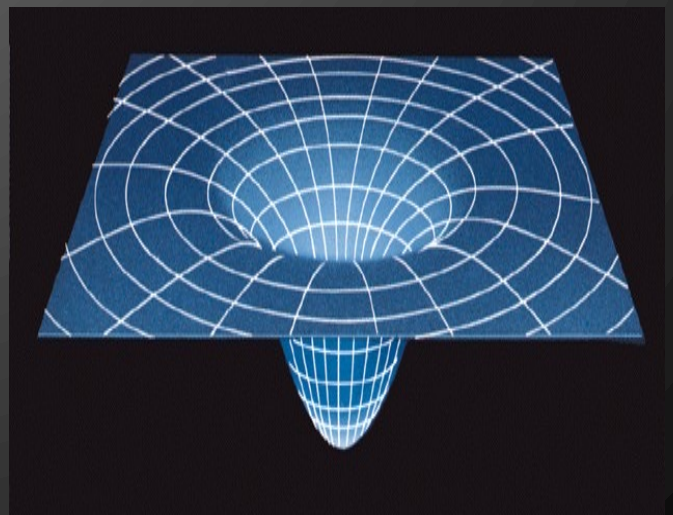
Dominate in accreting WD
(not relevant for BH)





Dominate in NS and BH

Release of gravitational binding energy



Accretion-, rotation-, thermal-, and magnetic field-powered X-ray sources

- Accretion-powered X-ray sources
  - Accretion of material from a companion star
    - ✓ white dwarf (CV, classical/dwarf novae, AM CVn, super-soft X-ray sources)
    - ✓ neutron star regular pulsed emission (HMXB)
    - ✓ neutron star X-ray burst sources (LMXB)
    - ✓ black hole soft X-ray transients (SXT)
    - ✓ continuous emission from hot accretion disks

Binaries

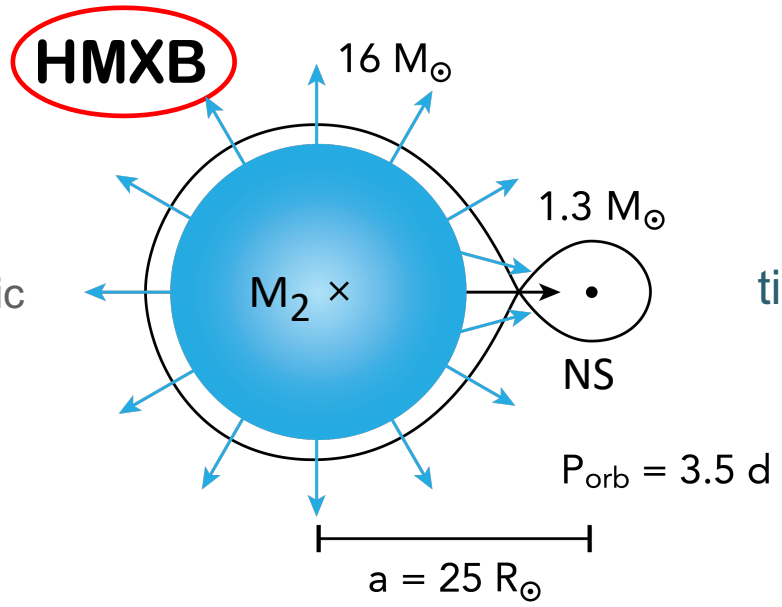
- Rotation-powered X-ray sources

Isolated NS

  - Crab pulsar, Geminga etc. (non-accreting X-ray pulsar)
- Thermal-powered X-ray sources
  - Continuous black body radiation from hot young neutron stars (XDINs)
- Magnetic-field powered X-ray sources
  - Magnetars (SGRs and AXPs)

High-Mass X-ray Binaries

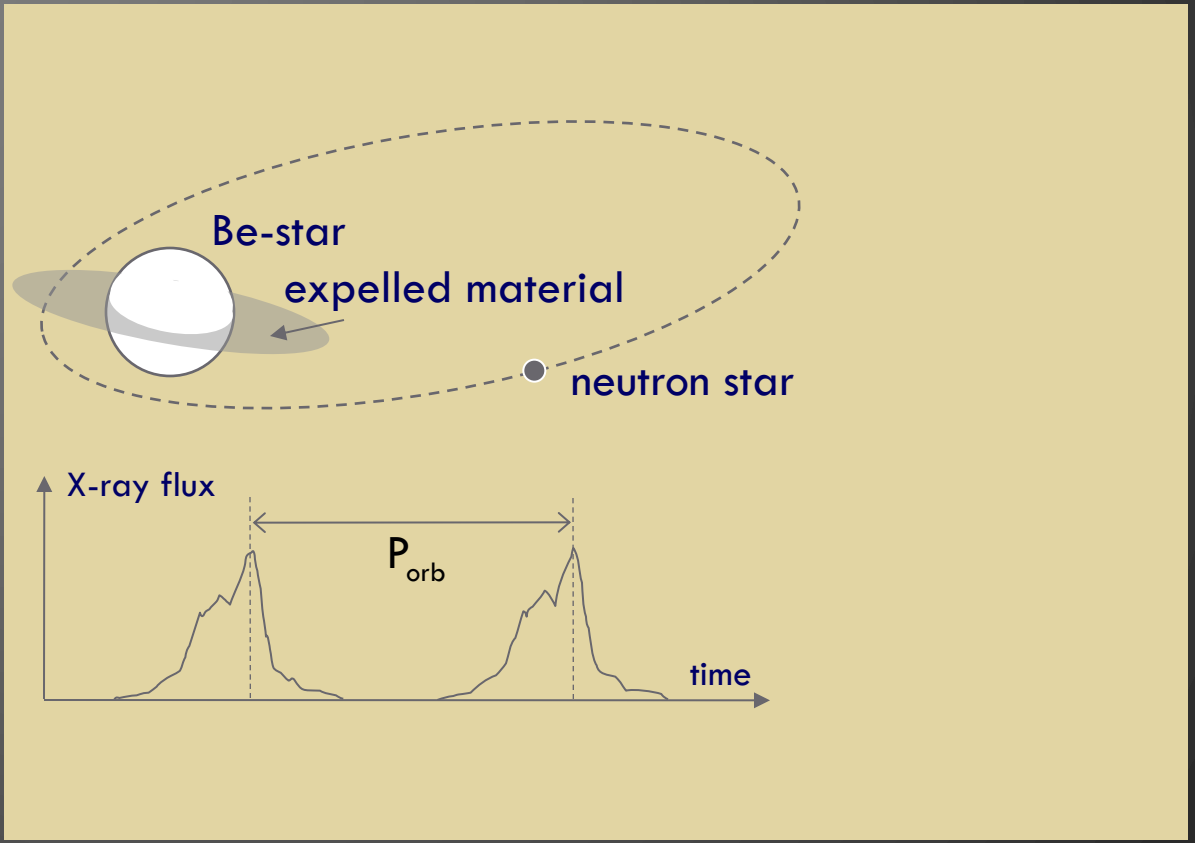
- wind accretion
- beginning atmospheric Roche-lobe overflow



timescale: 10^5 – 10^6 yr

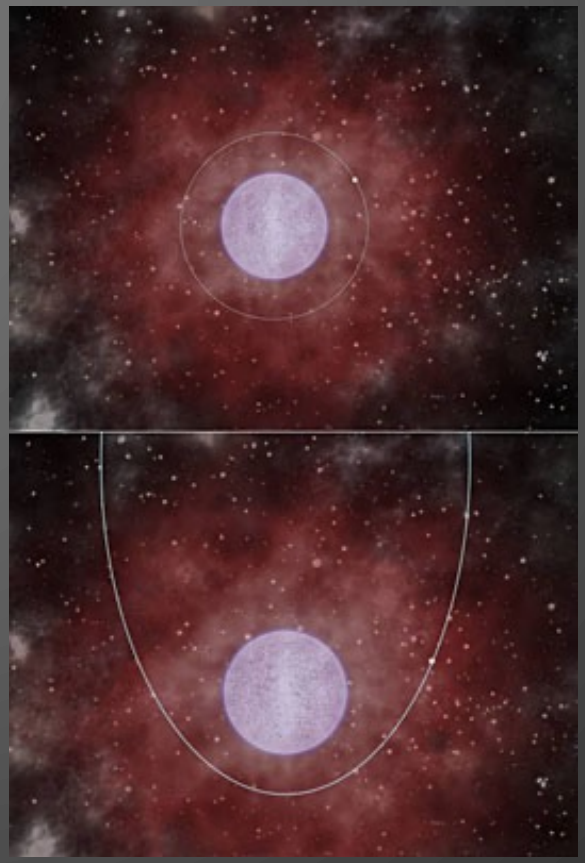
Donor star masses $M_2 > 10\ M_{sun.}$ NS or BH accretors.
114 systems known in our Galaxy:
- 50 systems with measured P_{orb} = 0.2 – 262 days (median 23 days)
- 66 systems with measured P_{spin} = 33 ms – 4 hours (median 3 min)

Be-star X-ray binary



Supergiant Fast X-ray Transients (SFXTs)

(subclass of HMXBs – many discovered with INTEGRAL)



HMXBs with BH accretors

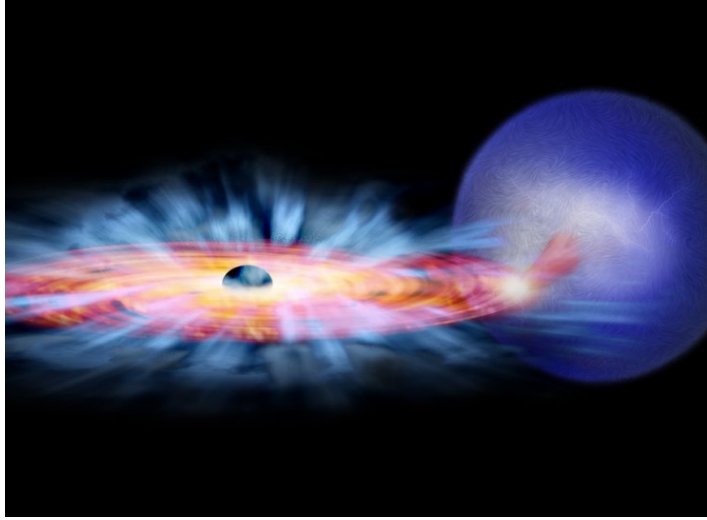
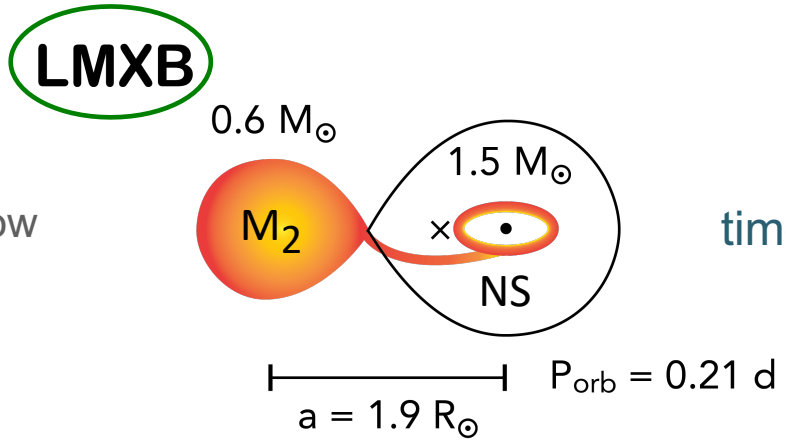


Table 6.6: The five known BH HMXBs. (Updated after van den Heuvel, 2019).

Source	$P_{\rm orb}$ (d)	$M_{\rm donor}$ ($M_\odot$)	$M_{\rm BH}$ ($M_\odot$)	Reference
Cyg X-1	5.6	41 ($\pm$7)	21.2 ($\pm$2.2)	Miller-Jones et al. (2021)
LMC X-1	3.9	31.8 ($\pm$3.5)	10.9 ($\pm$1.4)	Orosz et al. (2009)
LMC X-3	1.7	3.6 ($\pm$0.6)	7.0 ($\pm$0.6)	Orosz et al. (2014)
MCW 656	$\sim$ 60	$\sim$ 13	4.7 ($\pm$0.9)	Casares et al. (2014)
M33 X-7	3.45	70 ($\pm$7)	15.7 ($\pm$1.5)	Orosz et al. (2007)

Tauris & van den Heuvel (2023)

Low-Mass X-ray Binaries

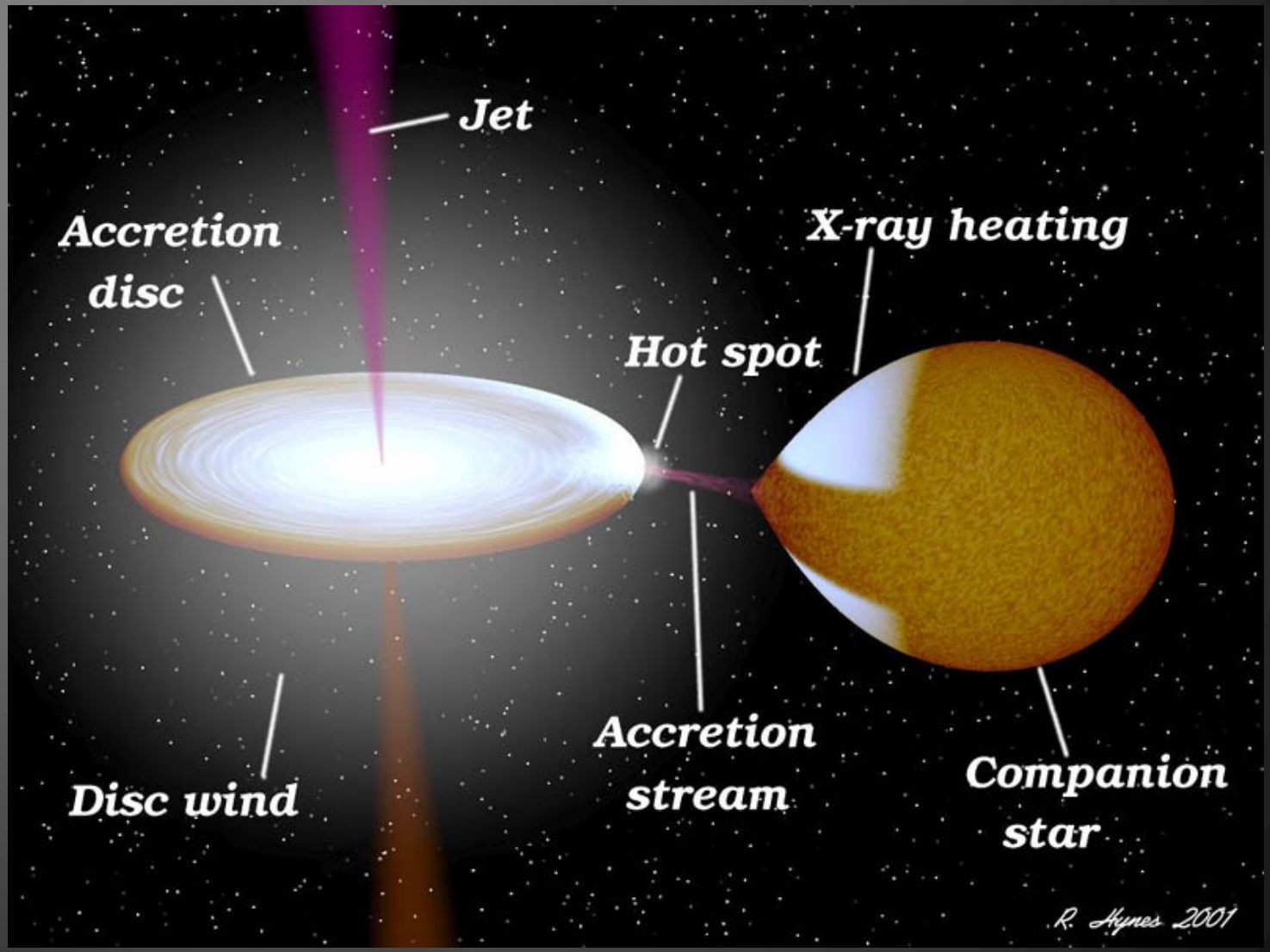


- Roche-lobe overflow

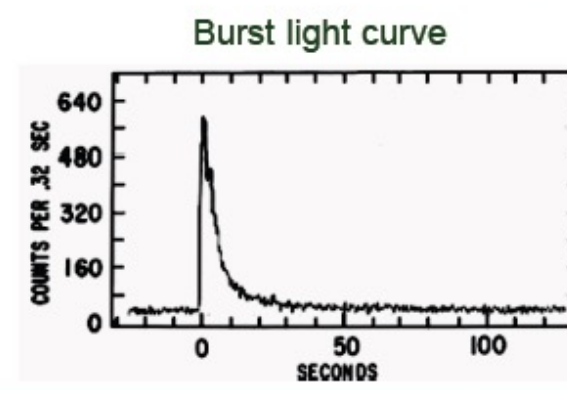
timescale: 10^8 – 10^9 yr

Donor star masses $M_2 < 1\ M_{sun.}$ NS or BH accretors.
187 candidate systems in our Galaxy:
- 74 systems with measured P_{orb} = 11 min – 1160 days (median: 8 hours)
- 26 systems with measured P_{spin} = 1.6 ms – 7.7 sec (median: 3 ms)

Most systems are **transients** and **X-ray bursters** (thermonuclear explosions)
About 20 **black hole** systems **(soft X-ray transients)**


Jet
Accretion disc
X-ray heating
Hot spot
Accretion stream
Companion star
Disc wind
R. Hynes 2001

X-ray bursts



Rise time ≈ 0.5 - 5 seconds
Decay time ≈ 10 - 100 seconds
Recurrence time ≈ hours to day
Energy release ≈ 10^{39} erg

Accretion of H-rich matter piles up on the NS.
Gravity compresses matter and the temperature rises at the base of this accumulated envelope leading to ignition under degenerate conditions resulting in a thermonuclear explosion and a rapid photospheric expansion.
The burst lasts for some 10-100 sec.
Burst oscillations reveal the spin period of the NS.

Equipotential surfaces

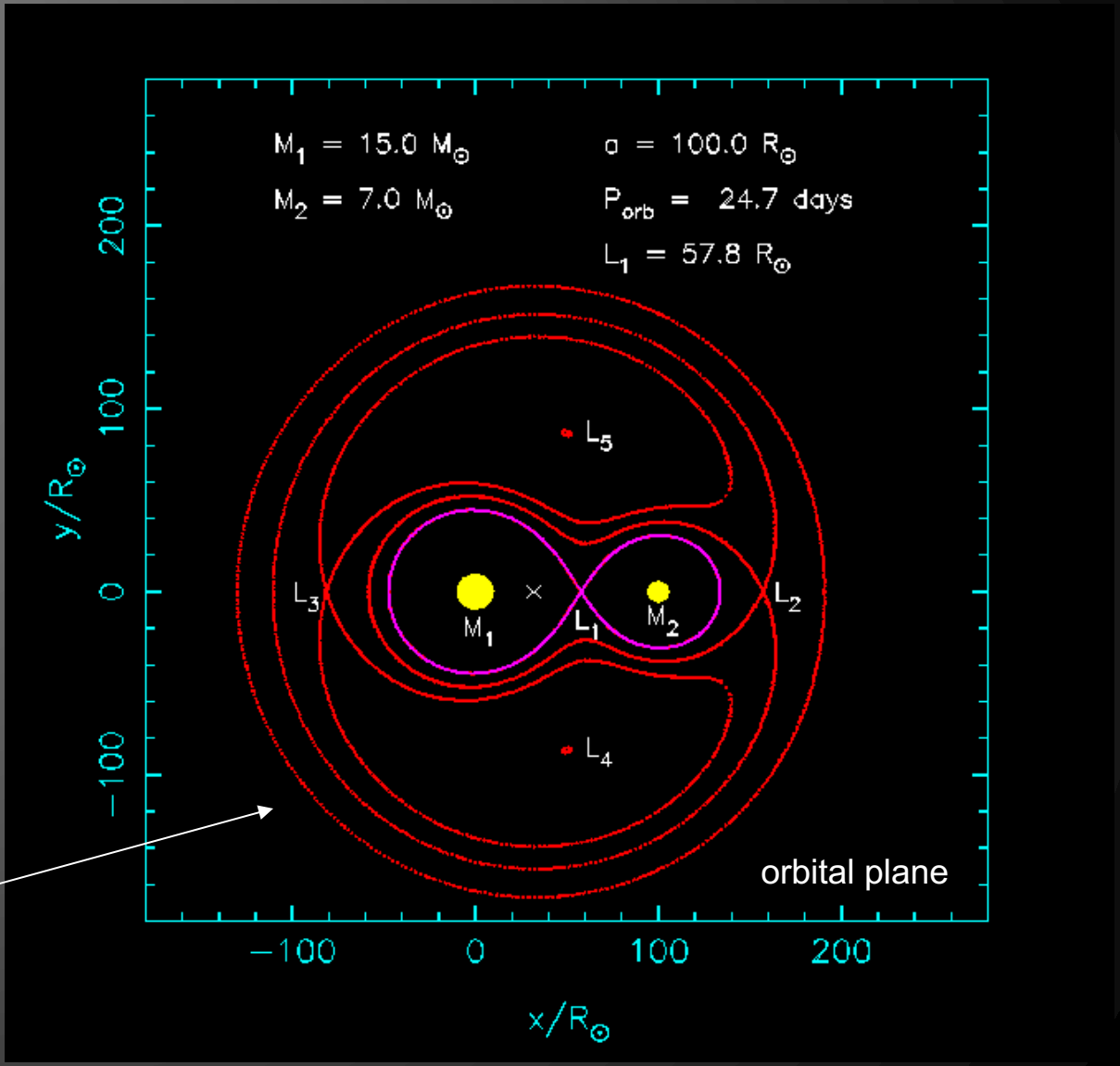
Effective gravitational potential:

$$\Phi = -\frac{GM_1}{r_1} - \frac{GM_2}{r_2} - \frac{\Omega^2 r_3^2}{2}$$

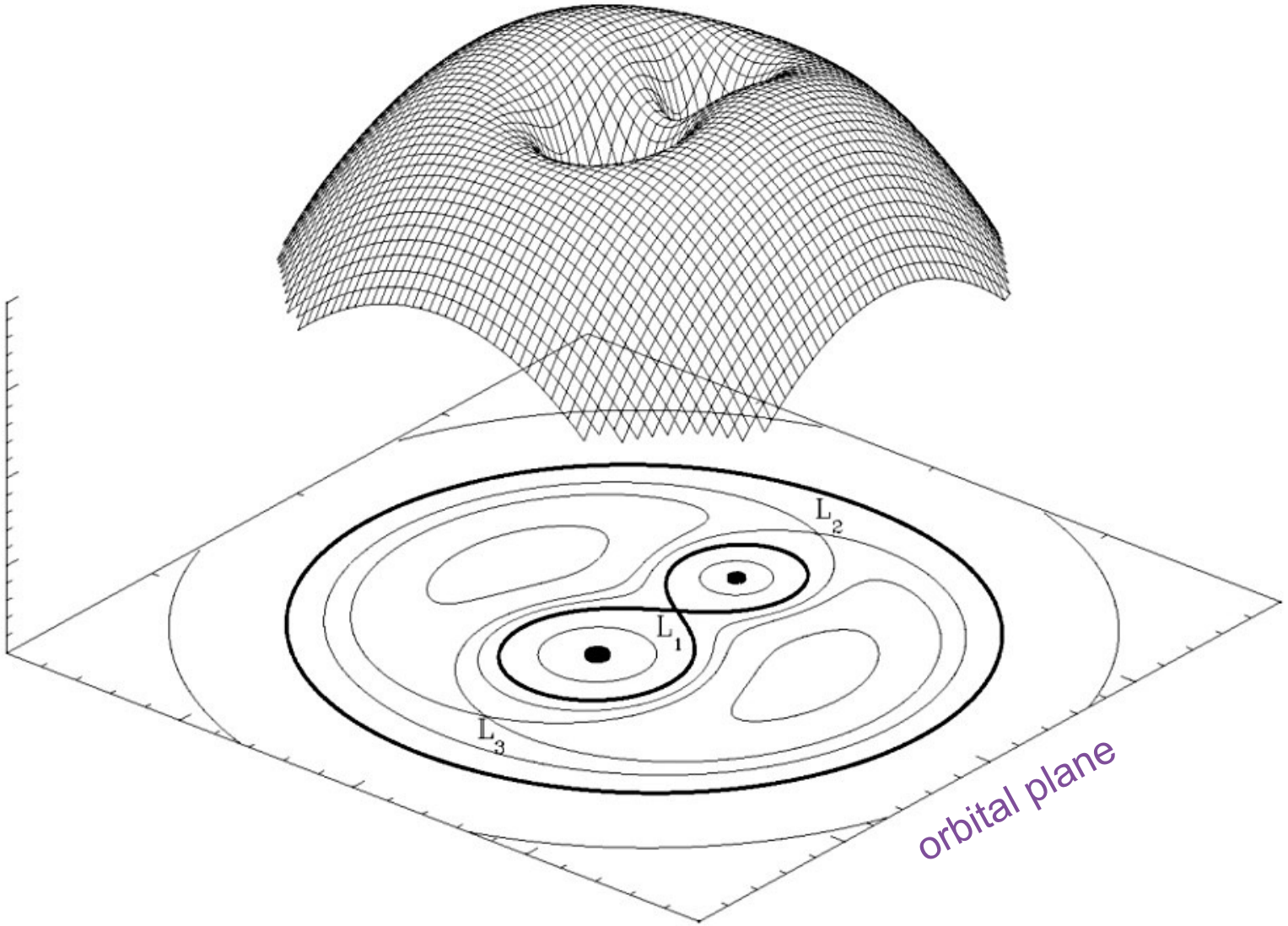
Roche-lobe radius:

$$\frac{R_L}{a} = \frac{0.49\,q^{2/3}}{0.6\,q^{2/3} + \ln\left(1 + q^{1/3}\right)}$$

co-moving frame



Equipotential surfaces



X-ray binaries Roche-lobe overflow Cases A, B and C:

(the evolutionary stage of the donor star at onset of RLO is quite important …)

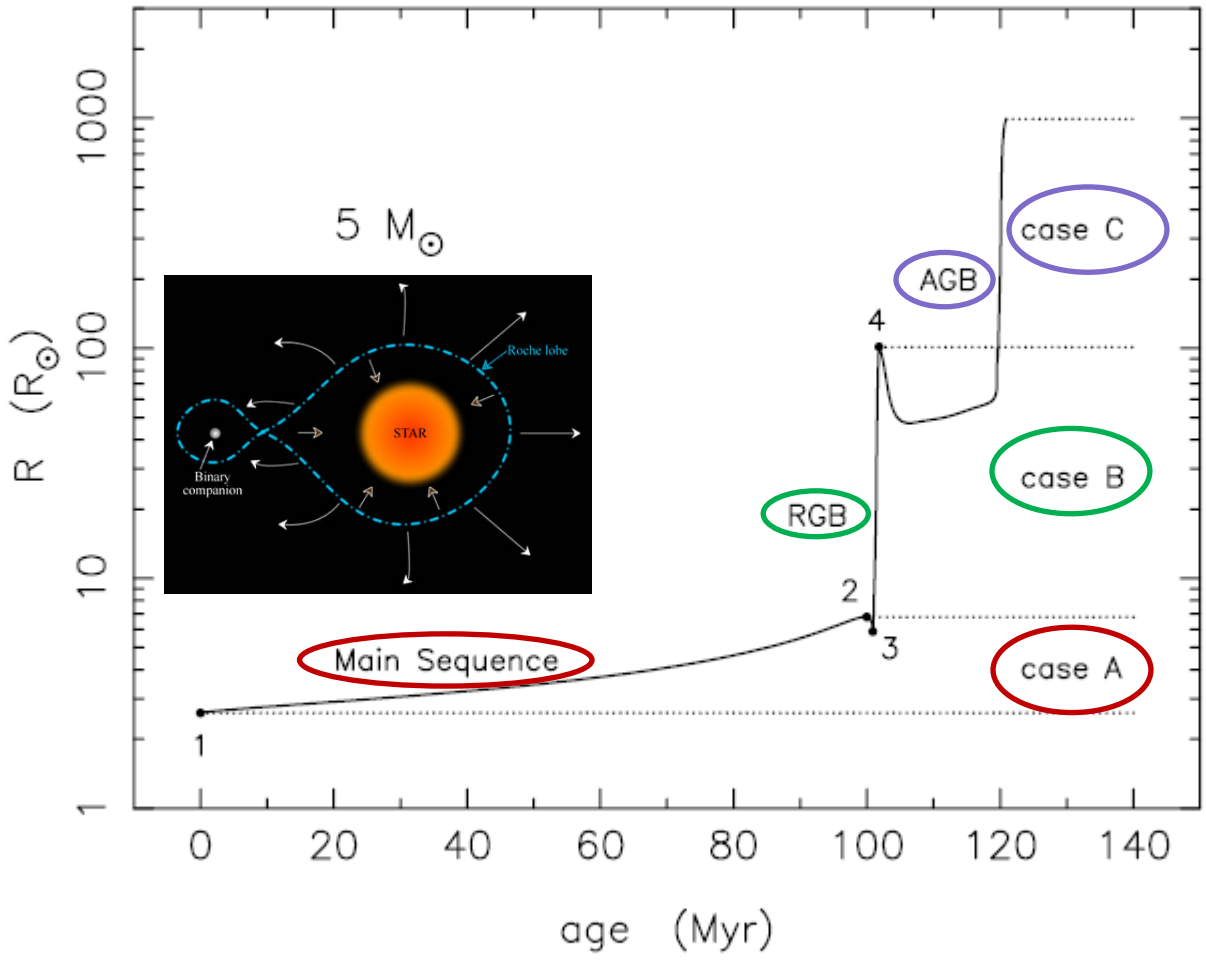


Fig. 16.6. Evolutionary change of the radius of the 5 $M_\odot$ star plotted in Fig. 16.5. The ranges of radii for mass transfer to a companion star in a binary system according to RLO cases A, B and C are indicated – see Section 16.4 for an explanation.

The evolution of compact binaries

Stability ?

response of donor star ?

response of Roche-lobe ?

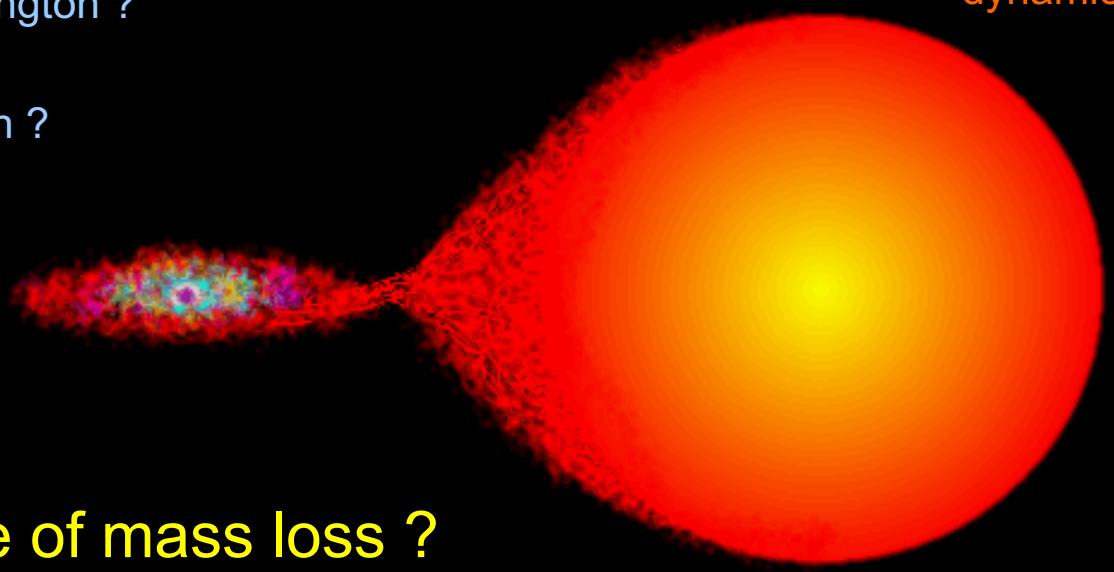
dynamically stable ?

Accretion ?

super-Eddington ?

jet ?

B-field, spin ?



Mode of mass loss ?

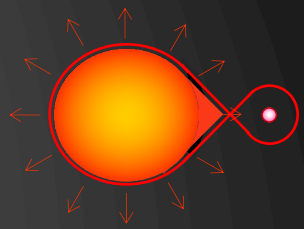
specific orbital angular momentum ?

magnetic braking / tidal interactions

gravitational wave radiation

Stability criteria for mass transfer

exponents of radius to mass: $R \propto M^{\zeta}$



$$\zeta_{donor} \equiv \frac{\partial \ln R_2}{\partial \ln M_2} \quad \wedge \quad \zeta_L \equiv \frac{\partial \ln R_L}{\partial \ln M_2}$$

adiabatic or thermal response of the donor star to mass loss

response of the Roche lobe to mass loss

initial stability criteria: $\zeta_L \leq \zeta_{donor}$

Change in stellar radius:

$$\dot{R}_2 = \left.\frac{\partial R_2}{\partial t}\right|_{M_2} + R_2 \, \zeta_{donor} \frac{\dot{M}_2}{M_2}$$

nuclear burning

Change in Roche-lobe radius:

$$\dot{R}_L = \left.\frac{\partial R_L}{\partial t}\right|_{M_2} + R_L \, \zeta_L \frac{\dot{M}_2}{M_2}$$

tidal spin-orbit couplings
gravitational wave radiation

$\dot{R}_2 = \dot{R}_L$ yields mass-transfer rate!

Stability criteria for mass transfer II

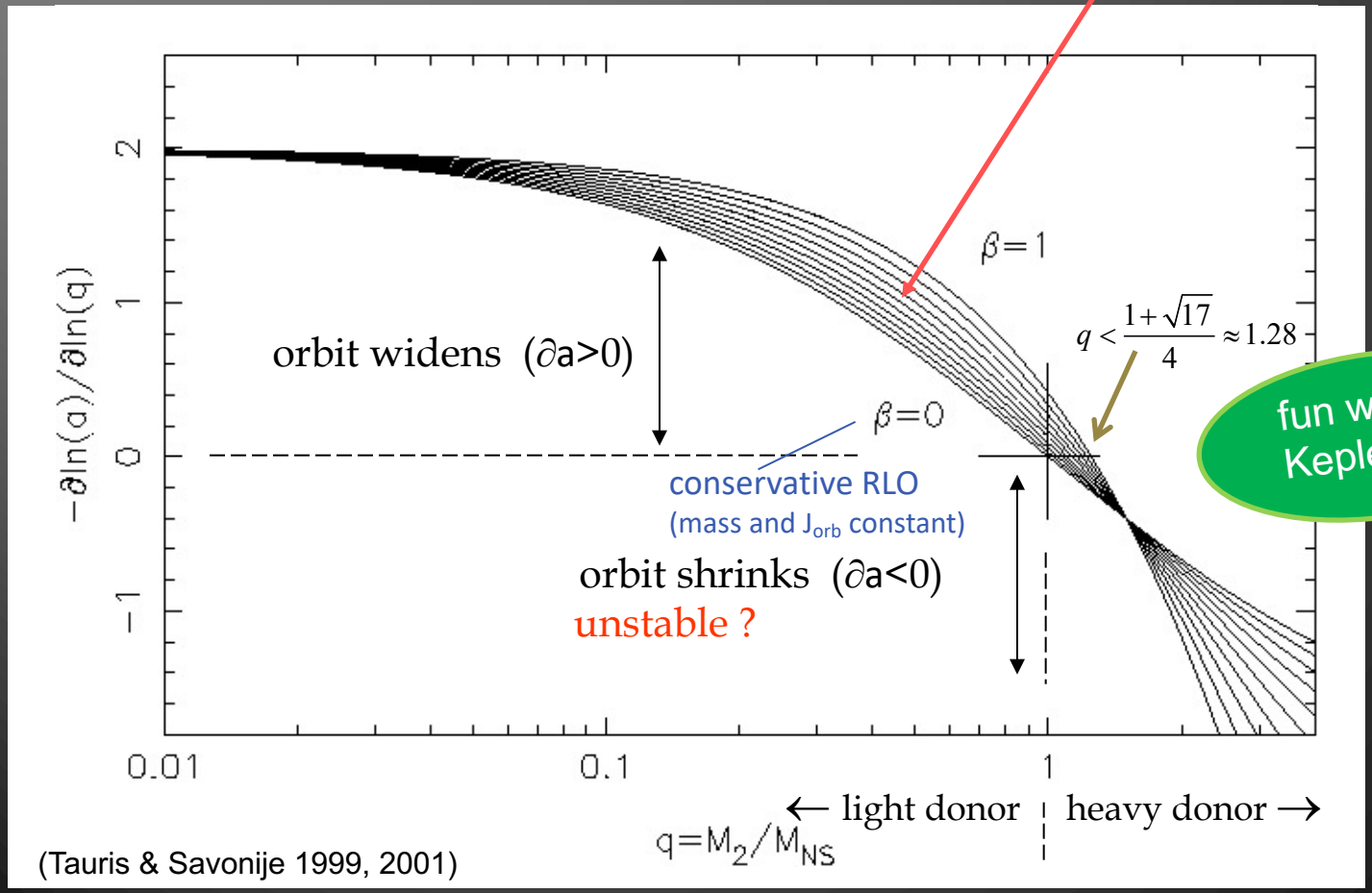
orbital separation, a
mass ratio, q

- Isotropic re-emission model

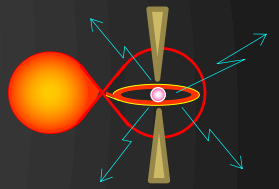
Orbital evolution:

$$-\frac{\partial \ln a}{\partial \ln q} \quad \wedge \quad q = \frac{M_2}{M_{NS}} \qquad (\partial q < 0)$$

$$\beta = \max\left(\frac{|\dot{M}_2| - \dot{M}_{Edd}}{|\dot{M}_2|}, 0\right) \quad \alpha = 0 \quad \delta = 0$$



(Tauris & Savonije 1999, 2001)



fun with Kepler 3.law

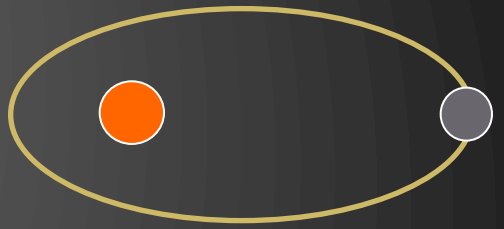
Summary

- Lighter donor: orbit widens
- Heavier donor: orbit shrinks

The Orbital Angular Momentum Balance Equation

$$J_{orb} = \frac{M_1 M_2}{M} \Omega a^2 \sqrt{1-e^2}$$

orbital angular momentum



logarithmic differentiation
(e=0, tidal circularization)

⇓

$$\frac{\dot{a}}{a} = 2\frac{\dot{J}_{orb}}{J_{orb}} - 2\frac{\dot{M}_1}{M_1} - 2\frac{\dot{M}_2}{M_2} + \frac{\dot{M}_1 + \dot{M}_2}{M}$$

$$J_{orb} = |\vec{r} \times \vec{p}|$$

$$\Omega = \sqrt{GM / a^3}$$

$$\frac{\dot{J}_{orb}}{J_{orb}} = \frac{\dot{J}_{gwr}}{J_{orb}} + \frac{\dot{J}_{mb}}{J_{orb}} + \frac{\dot{J}_{ls}}{J_{orb}} + \frac{\dot{J}_{ml}}{J_{orb}}$$

GW radiation

magnetic braking

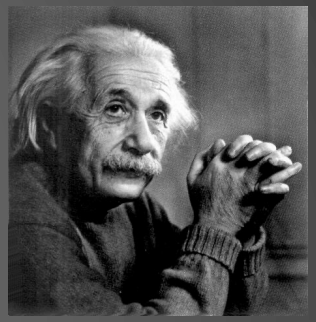
(other) spin-orbit couplings

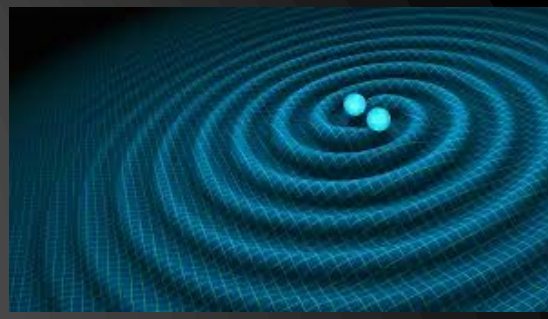
mass loss

see the next slides…

Gravitational wave radiation:

$$\frac{\dot{J}_{gwr}}{J_{orb}} = -\frac{32}{5}\frac{G^3}{c^5}\frac{M_1 M_2 M}{a^4}$$





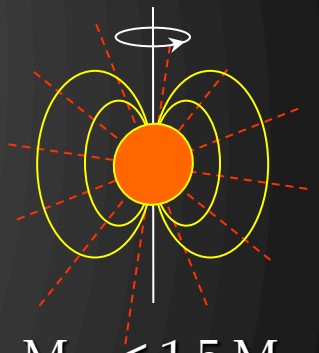
Magnetic braking:

$$\frac{\dot{J}_{mb}}{J_{orb}} \propto -\frac{k^2 R^4}{a^5}\frac{GM^3}{M_1 M_2}$$

(uncertain)

Low-mass stars: magnetic wind!
⇒ loss of spin angular momentum

In tight binaries the system
is tidally locked (synchronized)
and spin-orbit couplings operate
⇒ loss of orbital angular momentum



$M_2 < 1.5\ M_{sun}$

$P_{orb} < 2$ days

Spin-orbit couplings:

$$\frac{\dot{J}_{ls}}{J_{orb}}$$

- fx . change in stellar moment of inertia

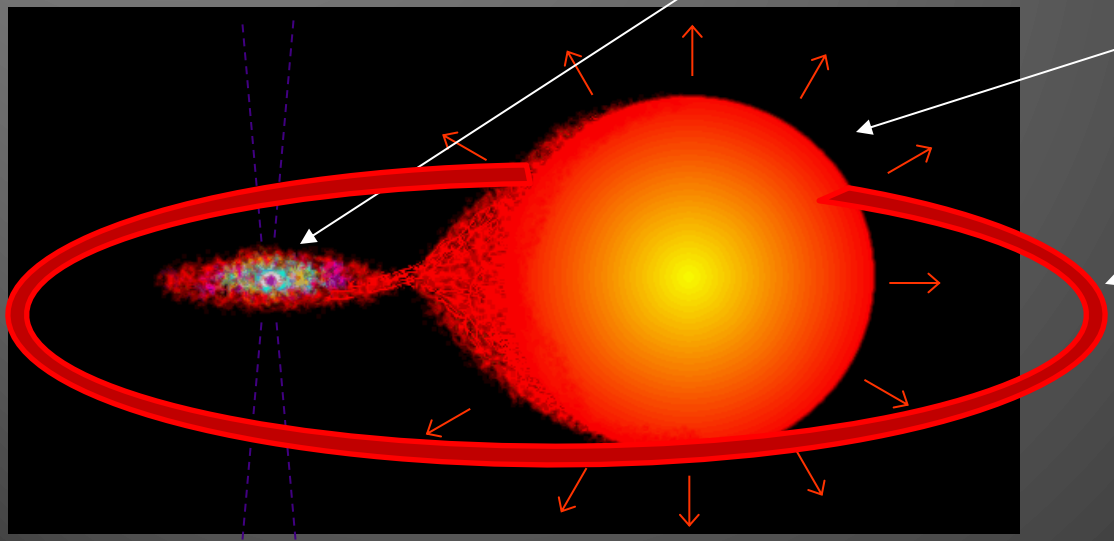
(as a result of nuclear burning or mass loss)

Isotropic re-emission model:

$$\beta = \max\left(\frac{|\dot{M}_2| - \dot{M}_{Edd}}{|\dot{M}_2|}, 0\right)$$

$$\frac{\dot{J}_{ml}}{J_{orb}} = \frac{\alpha + \beta q^2 + \delta\gamma(1+q)^2}{1+q}\frac{\dot{M}_2}{M_2}$$

β: mass ejected from accretor
- fx. in a relativistic jet
(isotropic re-emission)



α: loss in a direct fast wind

δ: mass loss via circumbinary coplanar toroid with radius: $\gamma^2 a$

$0 \leq (\alpha, \beta, \delta) \leq 1$ fractions of the material transferred from donor to accretor

accretion efficiency: $\varepsilon = 1 - \alpha - \beta - \delta$ $(\partial M_{NS} = -\varepsilon\, \partial M_2)$

Solution….

Integration of the OAMB eq. for mass transfer/loss:

$$\frac{a}{a_0} = \Gamma_{ls} \left(\frac{q}{q_0}\right)^{2(\alpha+\gamma\delta-1)} \left(\frac{q+1}{q_0+1}\right)^{\frac{-\alpha-\beta+\delta}{1-\varepsilon}} \left(\frac{\varepsilon q+1}{\varepsilon q_0+1}\right)^{3+2\frac{\alpha\varepsilon^2+\beta+\gamma\delta(1-\varepsilon)^2}{\varepsilon(1-\varepsilon)}}$$

Tauris & van den Heuvel (2006)

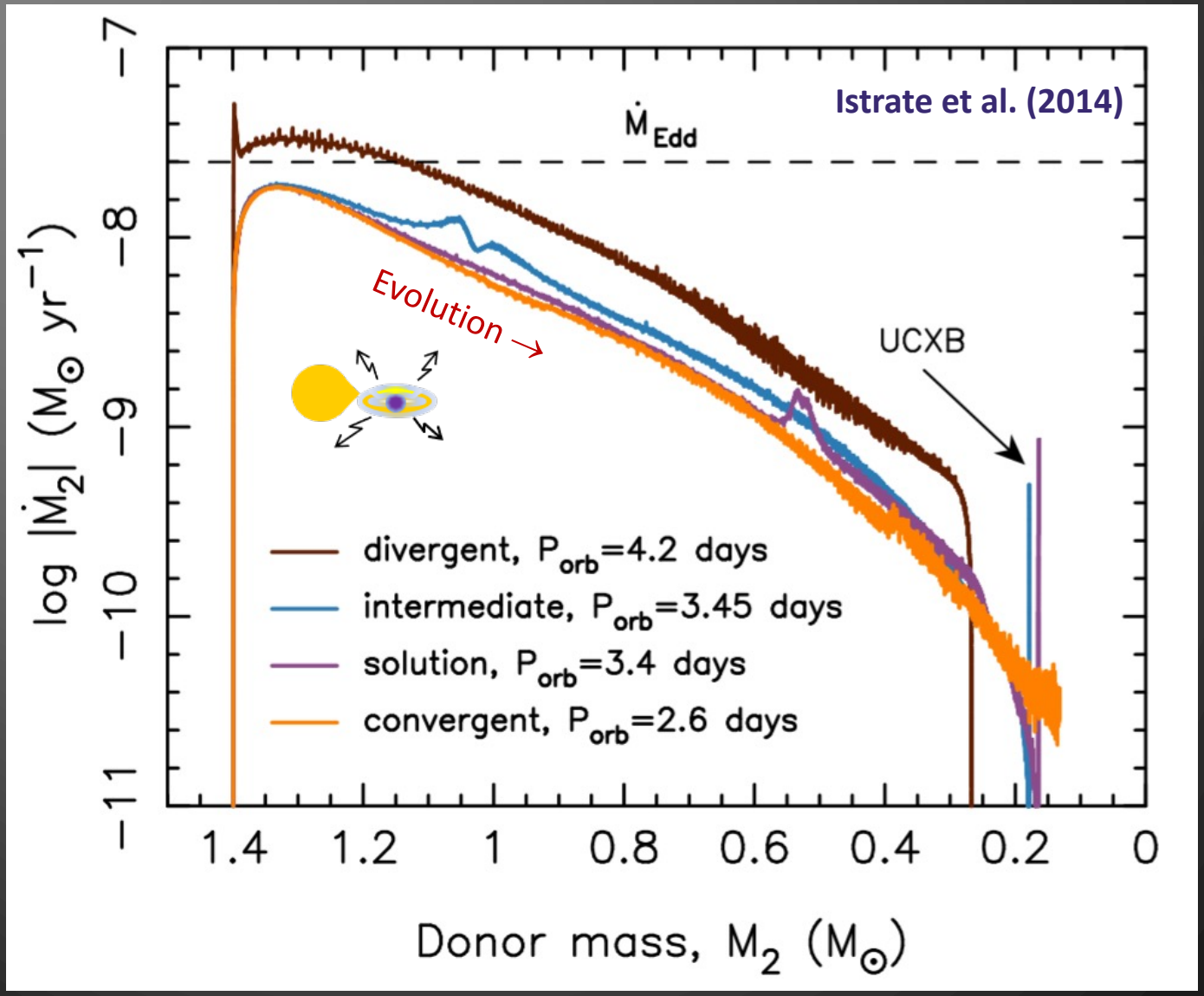
Eq.(4.58) in Tauris & van den Heuvel (2023)

mass ratio

final separation (after RLO)

For simpler cases with direct wind mass loss or conservative RLO: *Ex.9+10*

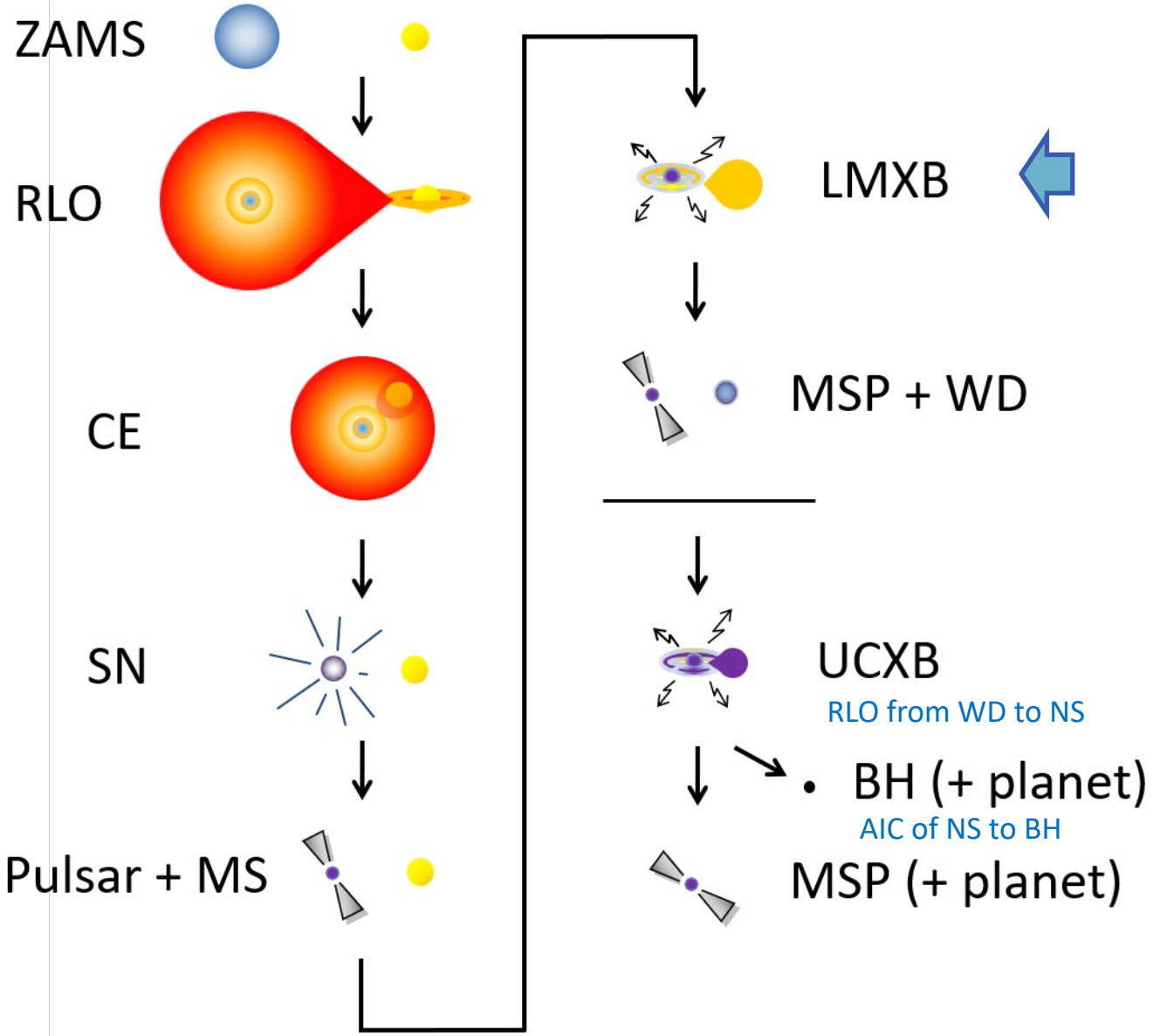
Example of numerical LMXB calculation……



LMXB

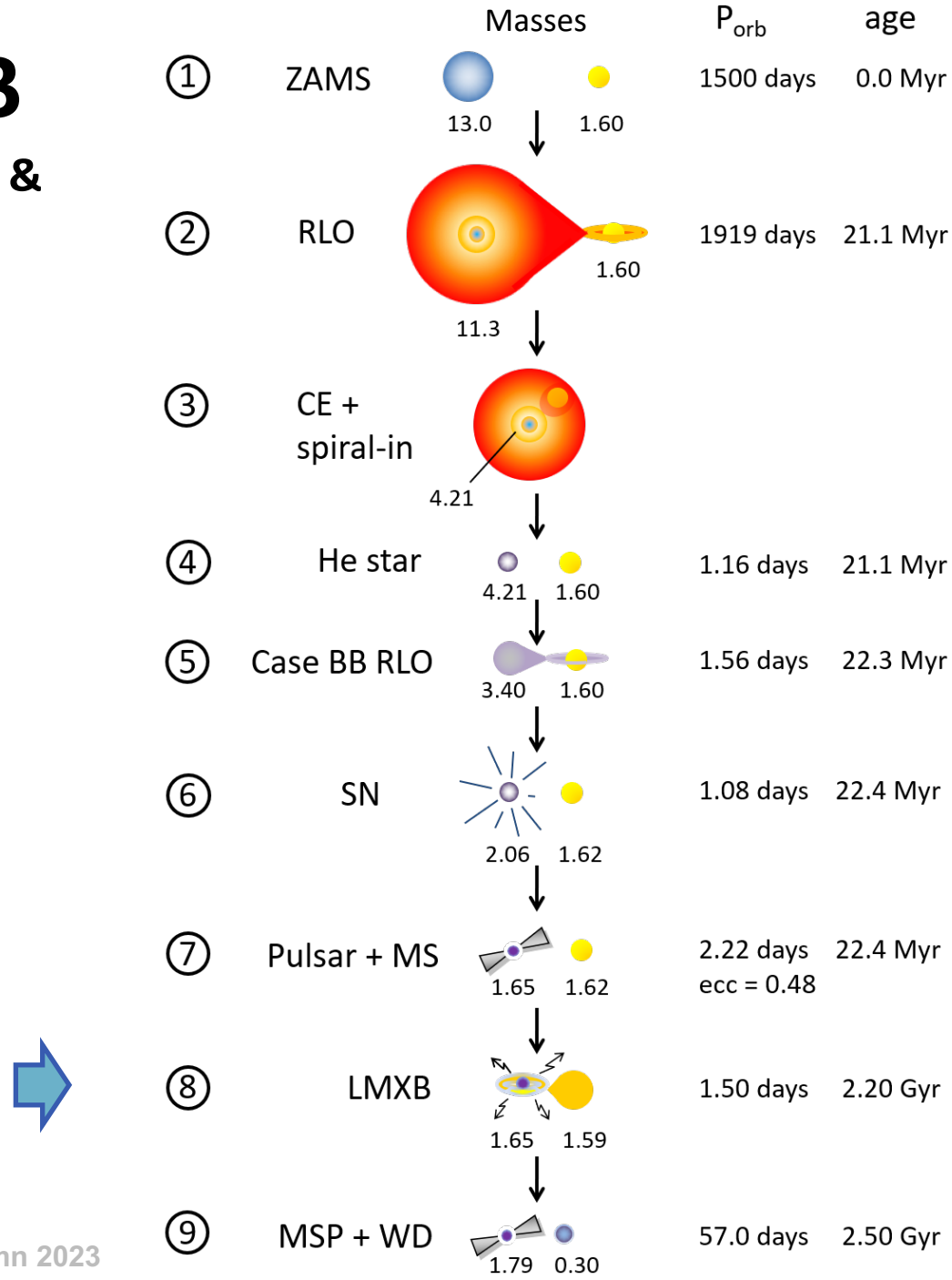
Formation & Evolution

Tauris & van den Heuvel (2023)



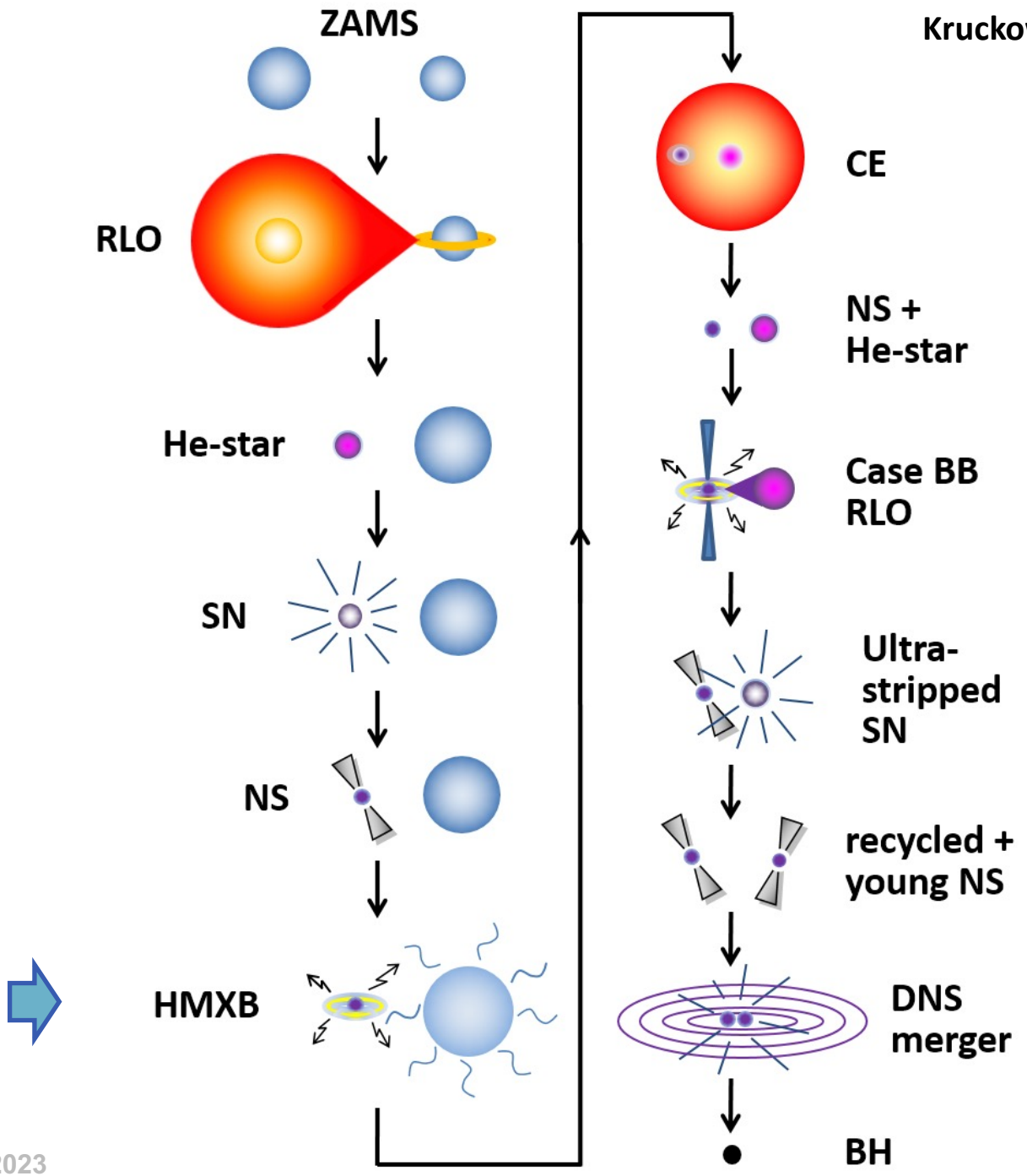
LMXB

Formation & Evolution in detail



HMXB

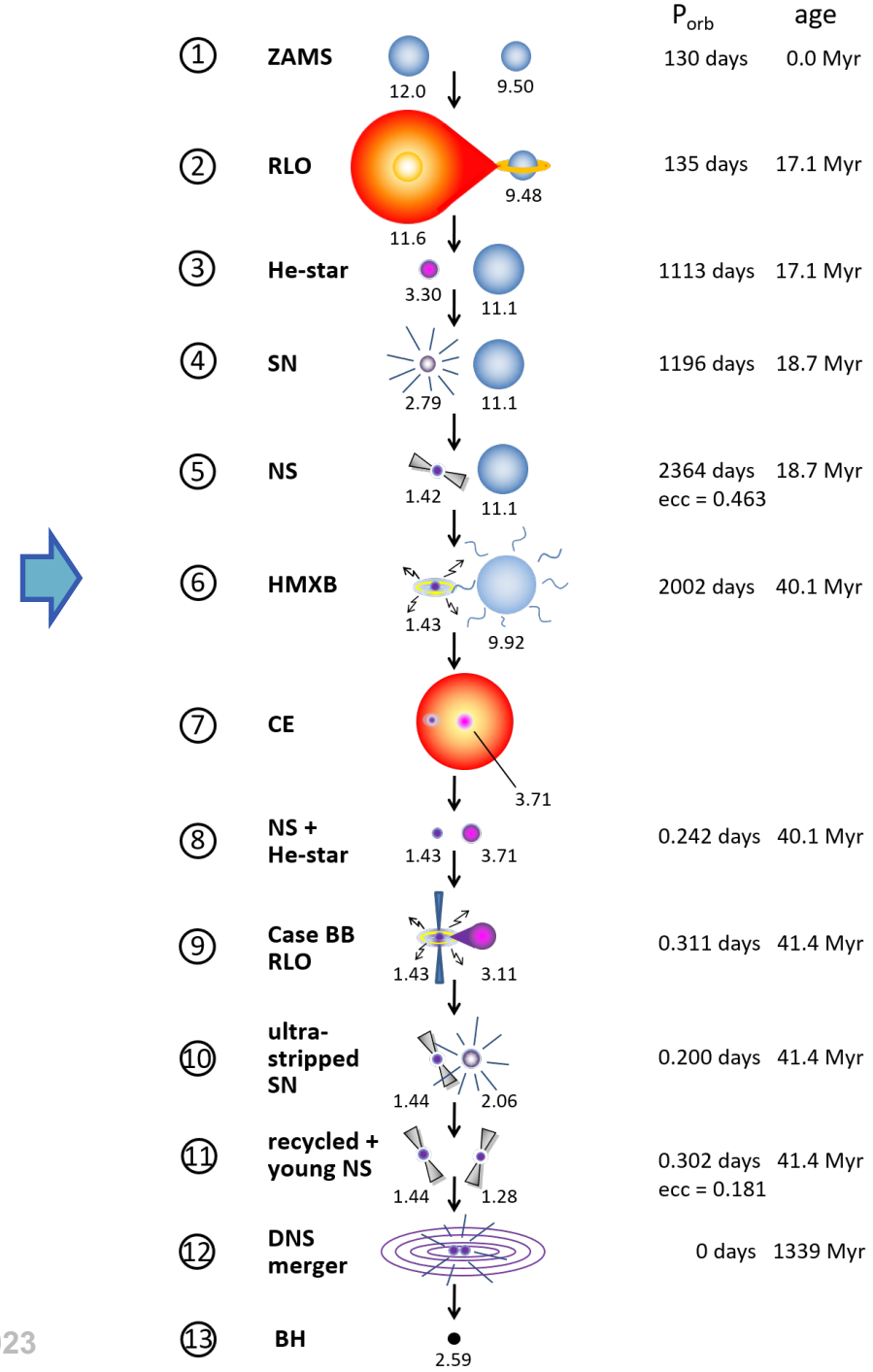
Formation & Evolution



Kruckow et al. (2018)

HMXB

Formation & Evolution in detail



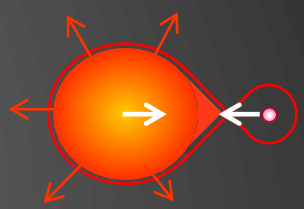
Tauris & van den Heuvel (2023)

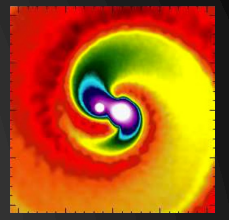
Common-Envelope + Spiral-in Evolution

Dynamically unstable mass transfer if:

- deep convective envelope of donor star
  = evolved giant star donors = wide orbits @ onset RLO
  (b/c rapid expasion in response to mass loss)
- $M_{donor} > M_{accretor}$

  (b/c orbit shrinks in response to mass loss)

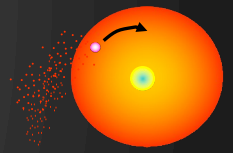




Run-away process!!

⇓

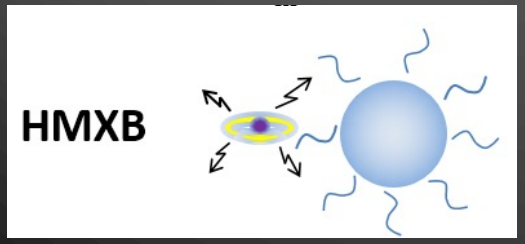
common envelope



drag force → dissipation of orb. ang. mom. + deposition of E_{orb} in the envelope

Outcome HMXBs → CE:

- CE: huge reduction of orbital separation



→ rejection of stellar envelope
(NS/BH orbiting a naked helium star)

→ merging of NS/BH + core
(Thorne-Zytkow object / BH)

COMMON-ENVELOPE EVOLUTION

$$\dot{E}_{orb} = -\frac{GM_{donor}M_{NS}}{2a^2}\frac{da}{dt} = \xi(\mu)\pi R_{acc}^2 \rho_{donor} v^3$$

Dissipation of E_{orb} by drag force (Bondi & Hoyle 1944)

$$E_{\rm env} \equiv \alpha_{\rm CE}\,\Delta E_{\rm orb}$$

Webbink (1984)

$$\alpha_{\rm CE} = 0.3 \sim 1$$

efficiency parameter

!

Energy of envelope:

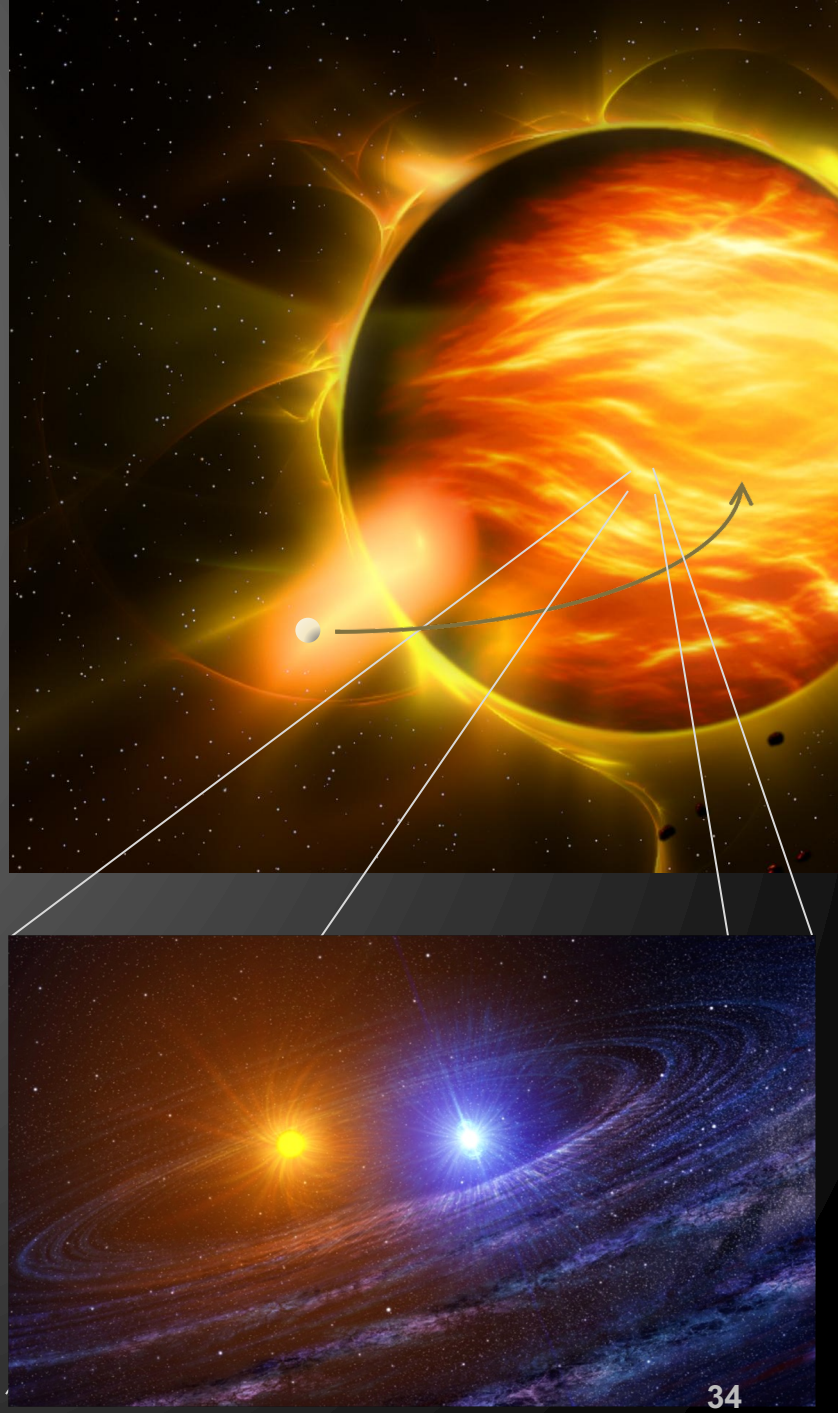
$$E_{env} = -\int_{M_{core}}^{M_{donor}} \frac{GM(r)}{r}\,dm \quad + \quad \alpha_{th}\int_{M_{core}}^{M_{donor}} U\,dm$$

gravitational binding energy

internal thermodynamic energy

Han et al. (1994, 1995)

- thermal energy
- energy of radiation
- ionization energy
- Fermi energy of e^--gas



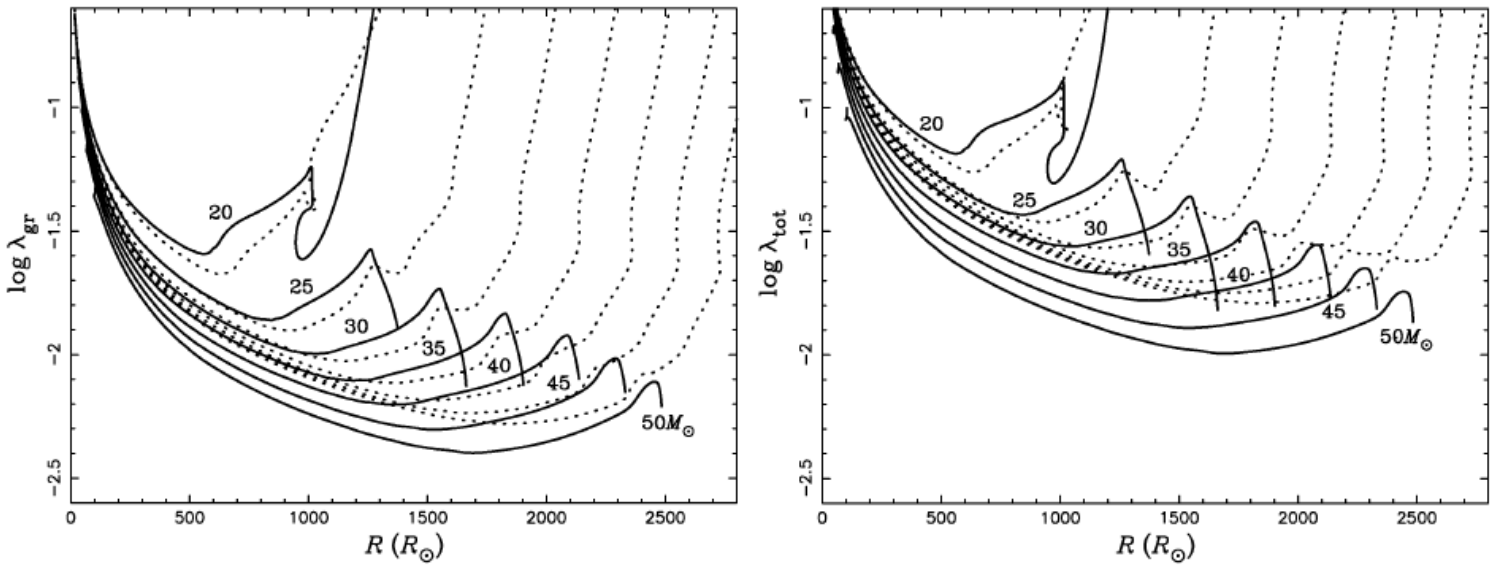


Figure 1. The envelope structure parameter λ as a function of stellar radius for different masses as indicated after hydrogen has been exhausted in the core. In the panels on the left, λ only includes the gravitational binding energy, while on the right λ includes both the gravitati… energy (similar to Dewi & Tauris (2000)). The dotted curves are calculated without inclusion of a stellar wind. Note that in this cas… largest radii attained by the models.

$$E_{env} \equiv -\frac{GM_{donor}M_{env}}{\lambda R_L}$$

de Kool (1990)

$$\lambda = 0.01 \sim 100$$

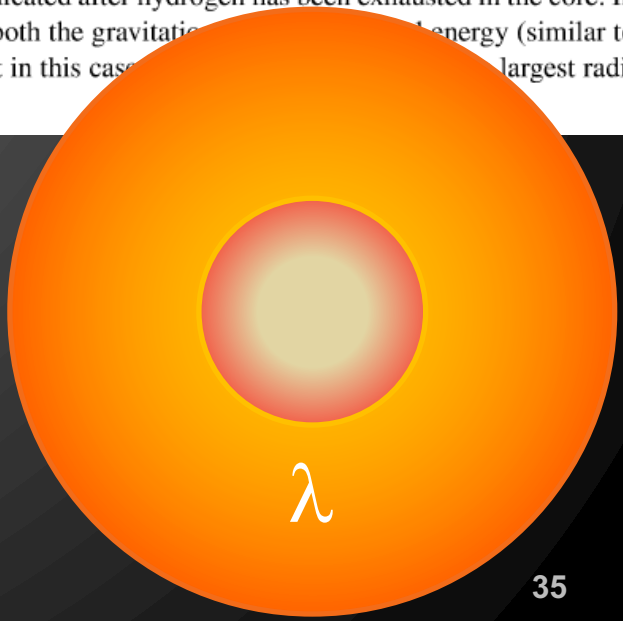
Dewi & Tauris (2000, 2001)

Podsiadlowski et al. (2003)

Xu & Li (2010)

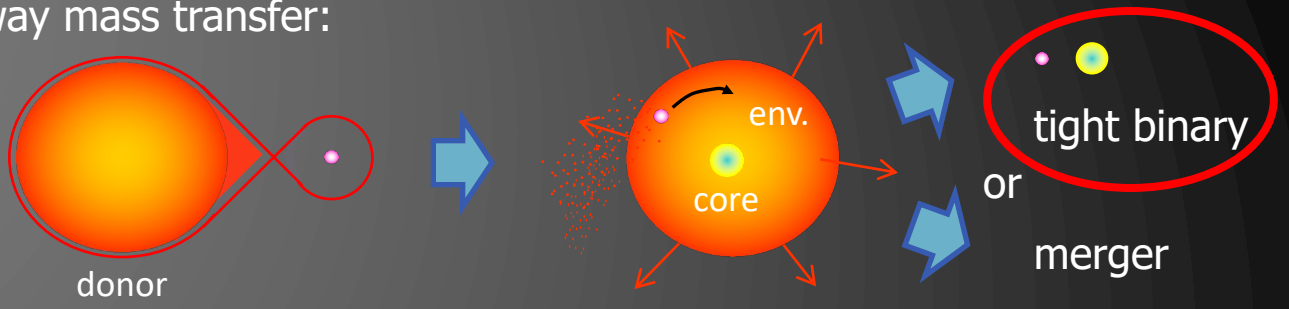
Loveridge et al. (2011)

Kruckow et al. (2016)



Common-Envelope + Spiral-in Evolution

Dynamically unstable run-away mass transfer:



$$E_{\rm env} \equiv \alpha_{\rm CE}\ \Delta E_{\rm orb} \qquad \alpha_{\rm CE} = 0.3 \sim 1$$

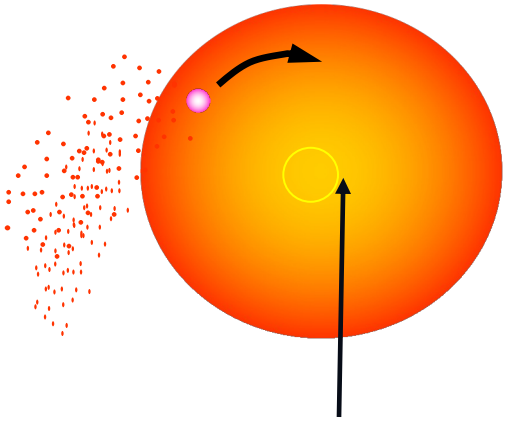
$$E_{env} \equiv -\frac{GM_{donor}\,M_{env}}{\lambda\, R_L} \qquad \Delta E_{orb} = -\frac{GM_{core}\,M_{NS}}{2a_f} + \frac{GM_{donor}\,M_{NS}}{2a_i}$$

$$\lambda = 0.01 \sim 100$$

$$\frac{a_f}{a_i} = \frac{M_{core}M_2}{M_{donor}}\,\frac{1}{M_2 + 2M_{env}/(\alpha_{\rm CE}\lambda r_L)} \Rightarrow \frac{a_f}{a_i} \approx 10^{-3} - 10^{-2}$$

Ex.11

COMMON ENVELOPE EJECTION

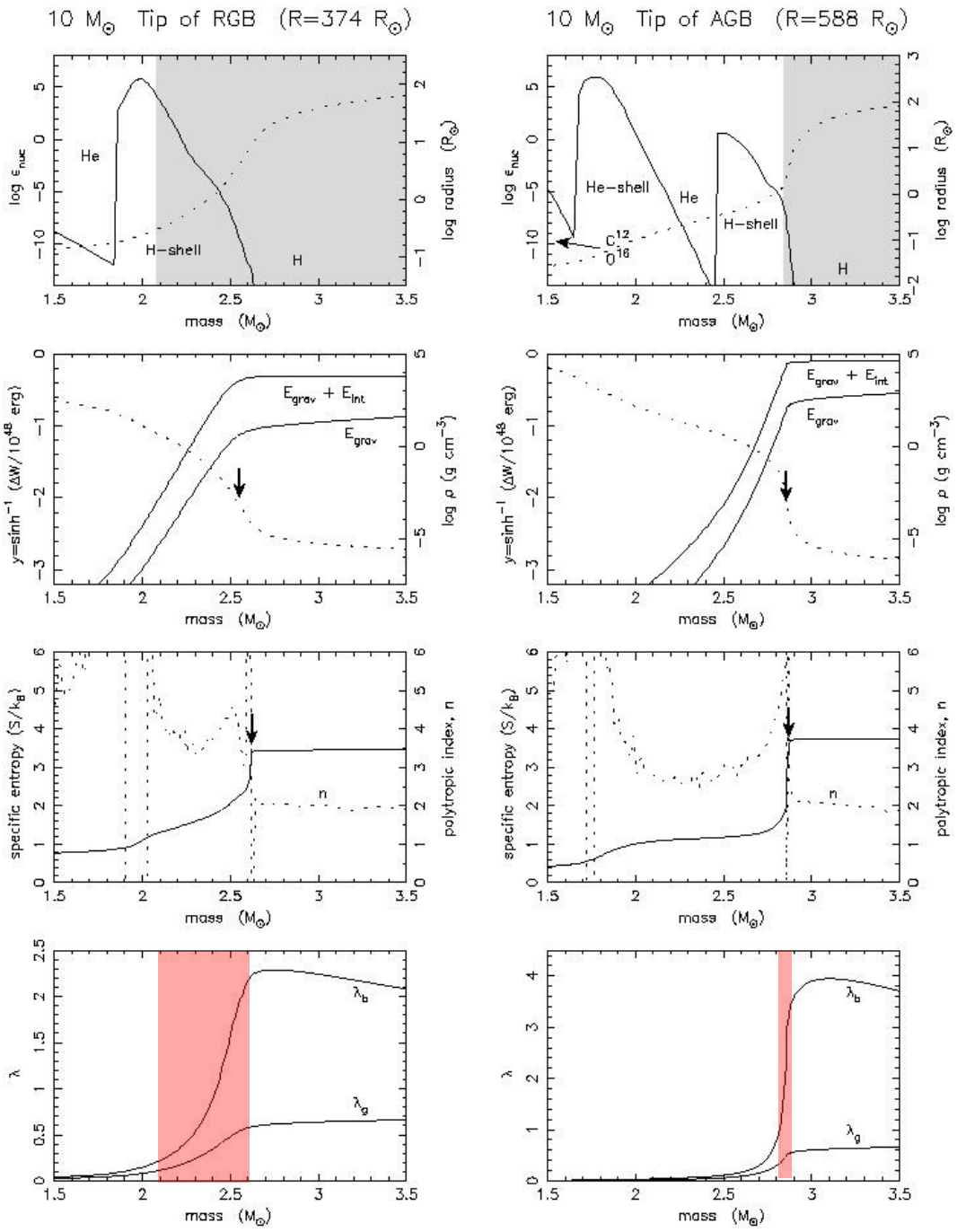


bifurcation point

Tauris & Dewi (2001)

Ivanova (2011)

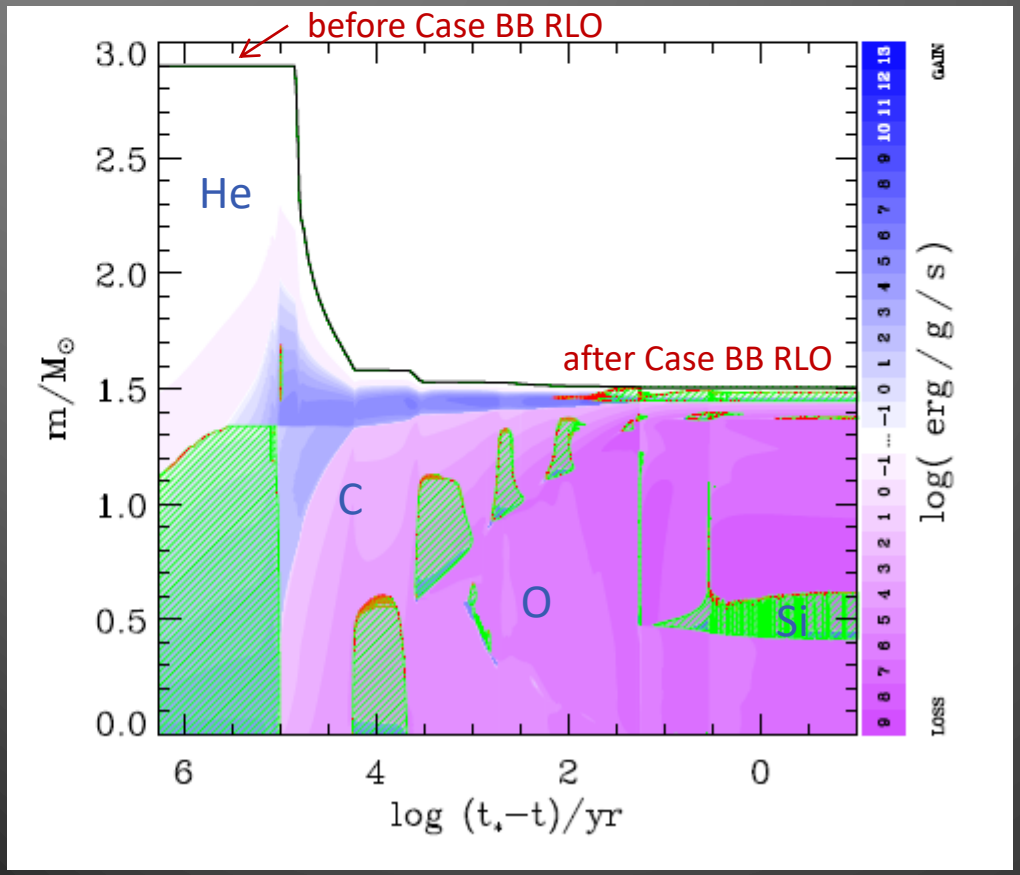
Kruckow et al. (2016)



Tauris & Dewi (2001)

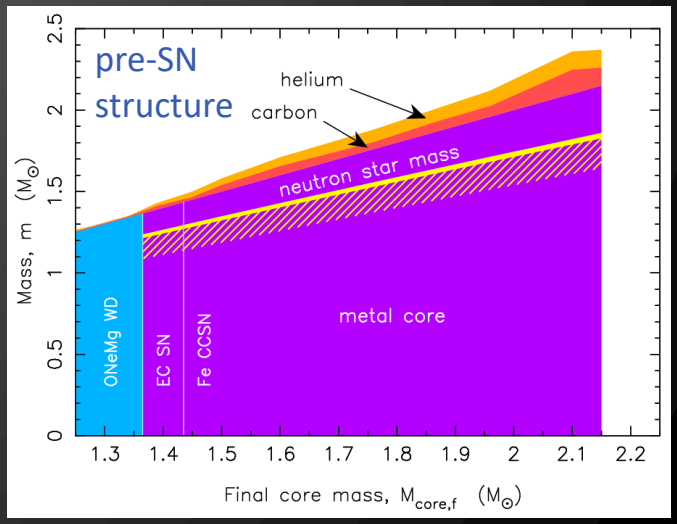
Ultra-stripped pre-SN metal core via Case BB RLO

Tauris et al. (2013)



Post-CE evolution of
He-star + NS in close orbit:
Case BB RLO strips off He env.
⇒ bare, pre-SN metal core just above Chandrasekhar mass

⇒ Iron core-collapse SN
with very little ejecta
+ formation of low-mass NS
in NS+NS system



Tauris et al. (2015)

Ultra-stripped pre-SN metal core via Case BB RLO

The Astrophysical Journal Letters, 920:L36 (6pp), 2021 October 20 https://doi.org/10.3847/2041-8213/ac2cc9

Novel Model of an Ultra-stripped Supernova Progenitor of a Double Neutron Star

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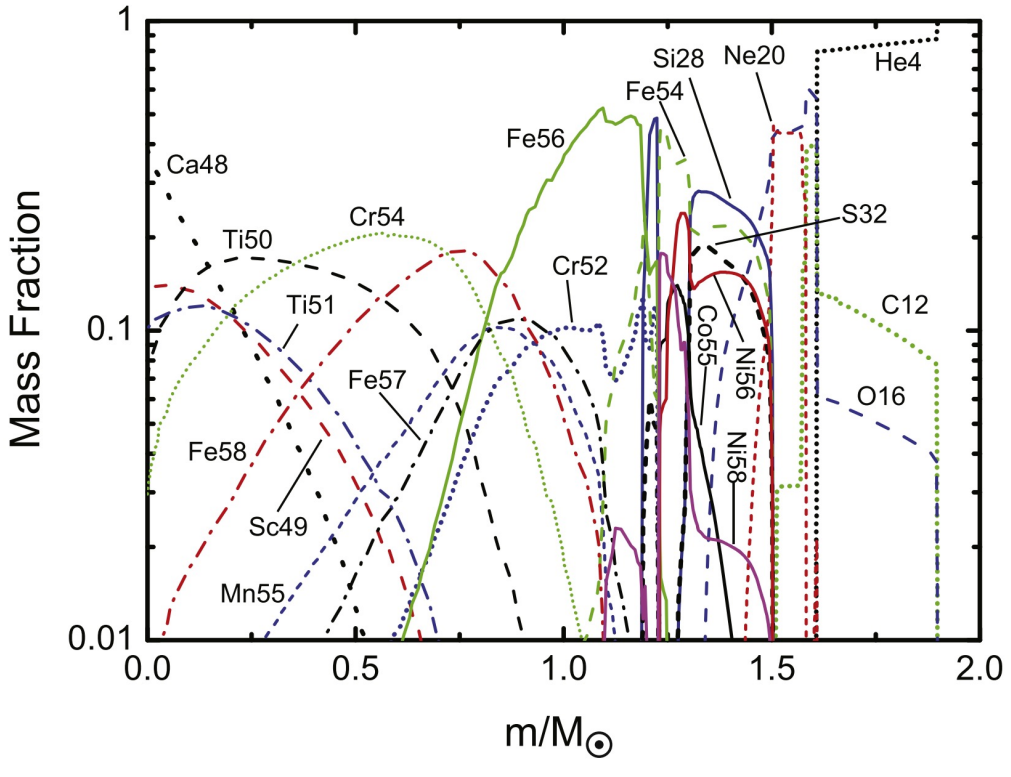
Received 2021 July 15; revised 2021 September 22; accepted 2021 October 5; published 2021 October 18

MESA



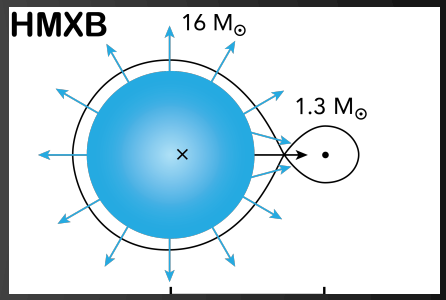
Intermediate-Mass X-ray Binaries

Why are so few IMXBs observed ?

HMXB: wind accretion (beginning atmospheric RLO)

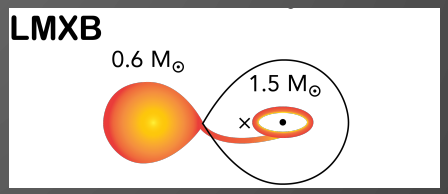
RLO is dynamically unstable and a CE forms

$$M_2 > 10\,M_\odot$$



LMXB: stable RLO

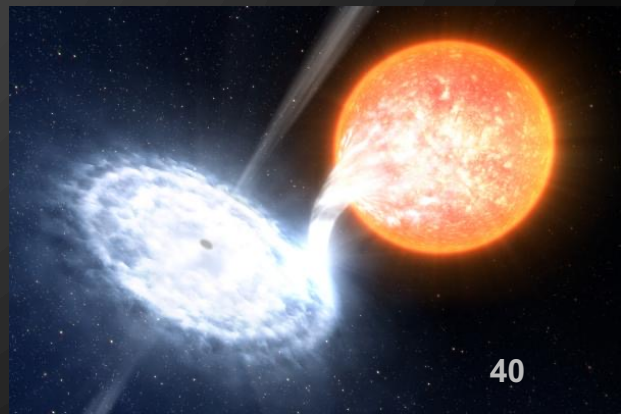
$$M_2 \leq 1.5\,M_\odot$$



IMXB: wind accretion is too weak, and

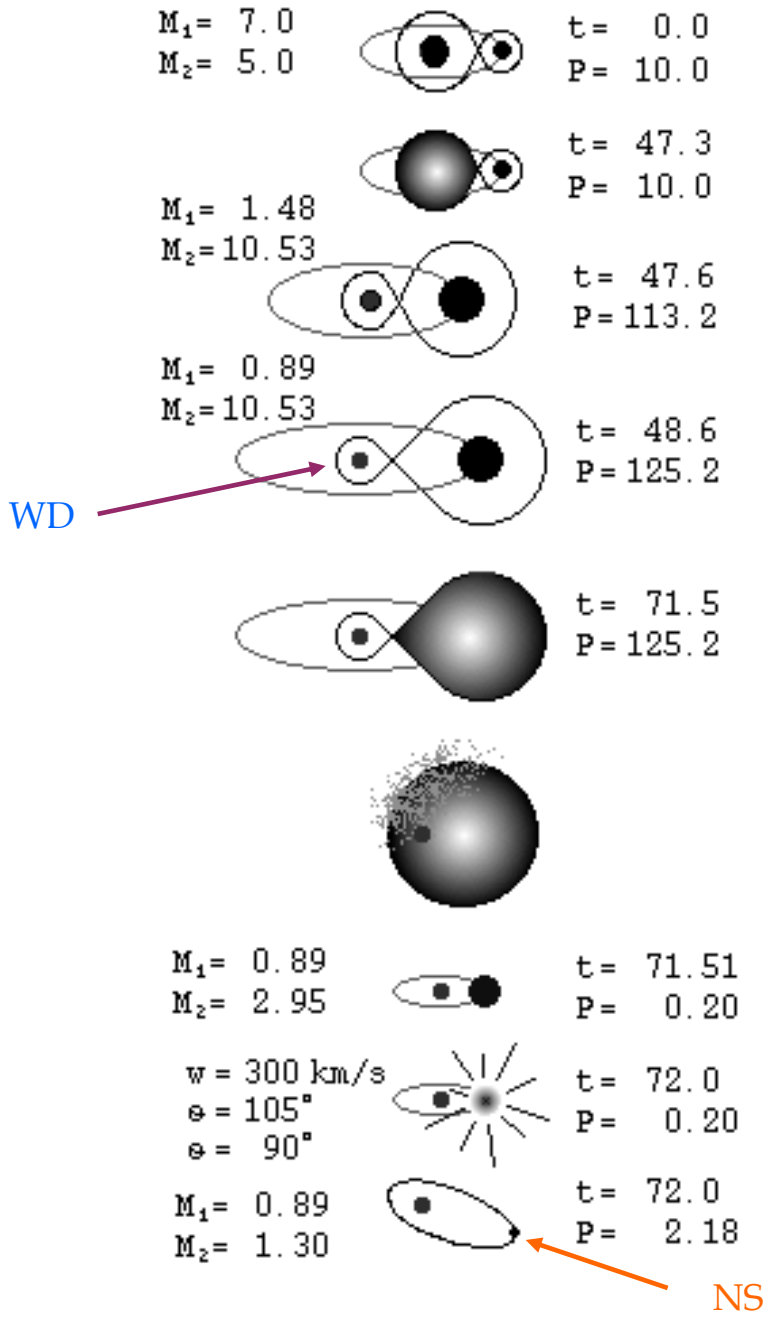
RLO is often unstable (or very short)

But evidence of post-IMXBs from pulsar+CO WD binaries



Double degenerate systems:

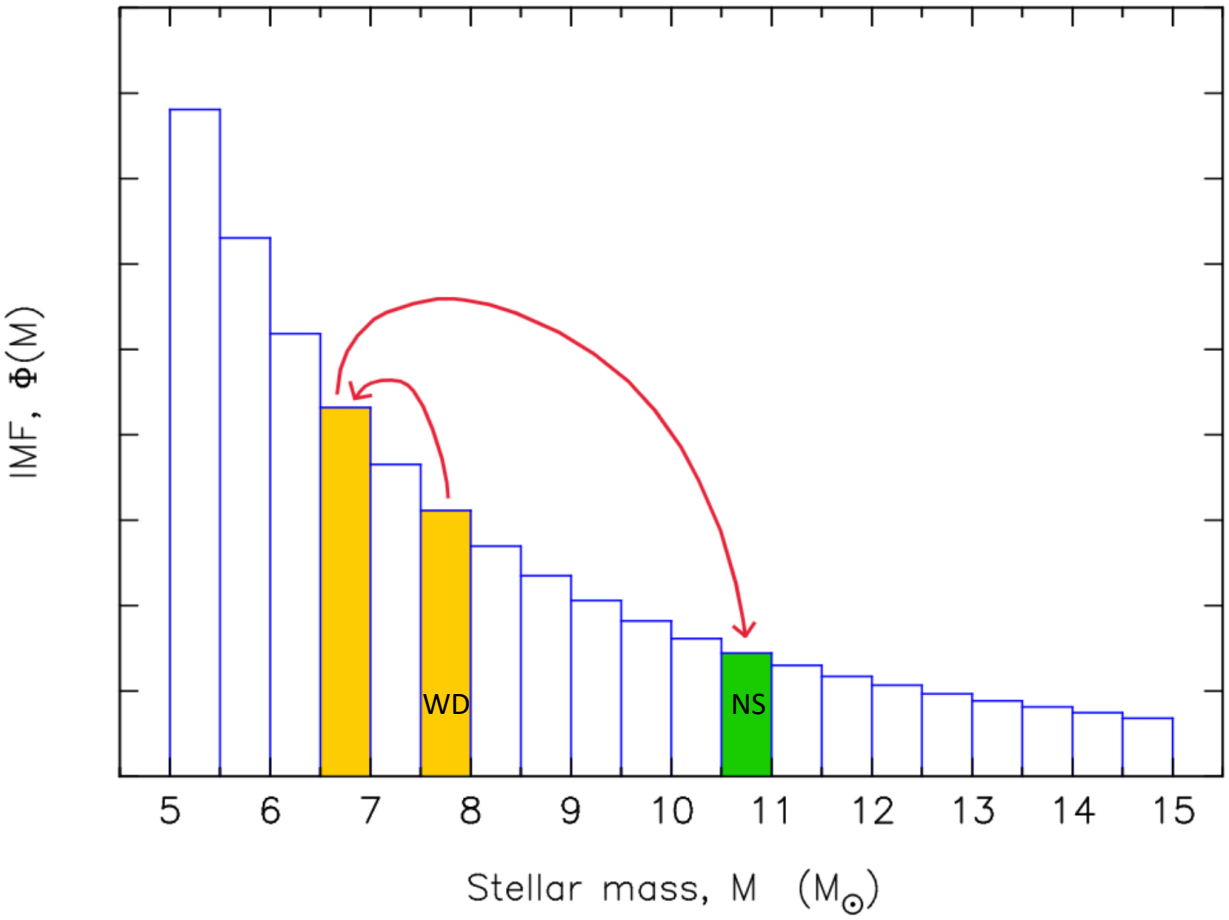
	observed systems	X-ray progenitor systems
WD+WD	(CO) WD + (CO) WD (AM CVn systems, RLO)	super-soft X-ray sources $M_2 > M_{WD}$ novae-like systems $M_2 < M_{WD}$
NS+WD	millisecond pulsars slow binary pulsars	LMXBs millisecond X-ray pulsars (SAX 1808.8-3658)
WD+NS	old WD + young pulsar (PSR 2303+46)	see next two slides with mass reversal
~~WD+BH~~	(not possible)	
NS+NS	recycled pulsar + young pulsar (PSR 1913+16, J0737-3039)	HMXBs (Vela X-1)
BH+WD	(no detections)	SXTs
BH+NS	BH + young pulsar (no detections)	HMXBs (Cyg X-1, LMC X-1)
NS+BH	semi-recycled pulsar + BH (no detections yet)	HMXBs (GX 301-2)
BH+BH	(only detectable via gwr)	HMXBs (MX33 X-7, IC10 X-1, NGC300 X-1)



Tauris & Sennels (1999)

Binaries undergoing mass reversal

Tauris & van den Heuvel (2023)



producing (WD+NS) systems

first formed

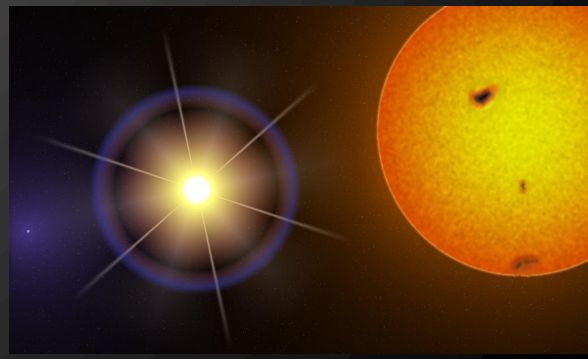
second formed
(non-recycled, young, eccentric orbit from SN)

Dynamical effects of asymmetric SNe

As a result of non-radial hydrodynamical instabilities newborn NSs receive a momentum kick at birth (resulting in a kick velocity of ~ 500 km s^{-1}).

The exact origin is still uncertain but is probably related to neutrino-driven convection bubbles, standing accretion shock instabilities (SASI) in the proto NS or other anisotropies in the ejecta which accelerate the proto-NS via the gravitational tug-boat mechanism, or simiply an asymmetric neutrino outflow (see Janka 2012 for a review).

- As a consequence of the imparted kick radio pulsars have large velocities (0-1000 km s^{-1}) and they have a wide scatter in their Galactic height distribution.
- Most (90%) of all potential LMXB systems are disrupted because of this kick.
- If SNe were purely symmetric then binary systems would be disrupted if $\Delta M/M > ½$ (as a consequence of the virial theorem). Thus a kick can also help to keep systems bound if the direction of the kick is towards the companion star.



Dynamical effects of asymmetric SNe

is:

$$\frac{X_+}{A} = -\cos\beta\left[\xi\sin\gamma + (\xi-1)\sqrt{\frac{\xi}{\xi-2}}\right] \quad (38)$$

$$\frac{Y_+}{A} = \xi\cos^2\gamma - 1 - \sin\gamma\sqrt{\frac{\xi}{\xi-2}} \quad (39)$$

$$\frac{Z_+}{A} = -\sin\beta\sin\lambda\left[\xi\sin\gamma + (\xi-1)\sqrt{\frac{\xi}{\xi-2}}\right] \quad (40)$$

where we have used $u_\infty = A\,e^2$ and $u_0 = u_\infty\sqrt{\xi/(\xi-2)}$.

We now proceed to express β, γ and λ in the true input angles ϑ and φ. We cannot reach $\sin\lambda$ directly, but that doesn't matter, from Fig. 1 (bottom) we have: $u_0\sin\beta\sin\lambda = w\sin\vartheta\sin\varphi$. Intermediate results are:

$$X_+ = \frac{v + w\cos\vartheta}{1-\xi+\sqrt{\xi(\xi-2)}\sin\gamma} \quad (41)$$

$$Y_+ = \frac{\sqrt{\xi(\xi-2)}}{1+\xi(\xi-2)\cos^2\gamma}\times \left[u_0\left(1-\frac{1}{\xi}\right) - \frac{1}{u_0}(w\sin\vartheta\cos\varphi - v_{\rm im})^2\right] - \frac{(w\sin\vartheta\cos\varphi - v_{\rm im})}{1+\xi(\xi-2)\cos^2\gamma} \quad (42)$$

$$Z_+ = \frac{w\sin\vartheta\sin\varphi}{1-\xi+\sqrt{\xi(\xi-2)}\sin\gamma} \quad (43)$$

To shorten the expressions a little more, define:

$$P \equiv 1 - 2\tilde{m} + \frac{w^2}{v^2} + \frac{v_{\rm im}^2}{v^2} + 2\frac{w}{v^2}(v\cos\vartheta - v_{\rm im}\sin\vartheta\cos\varphi) \quad (44)$$

$$Q \equiv 1 + \frac{P}{\tilde{m}} - \frac{(w\sin\vartheta\cos\varphi - v_{\rm im})^2}{\tilde{m}v^2} \quad (45)$$

$$R \equiv \left(\frac{\sqrt{P}}{\tilde{m}v}(w\sin\vartheta\cos\varphi - v_{\rm im}) - \frac{P}{\tilde{m}} - 1\right)\frac{1+m_{2f}}{m_{2f}} \quad (46)$$

$$S \equiv \left(1 + \frac{P}{\tilde{m}}(Q+1)\right)\frac{1+m_{2f}}{m_{2f}} \quad (47)$$

Inserting Eqs. (48)–(50) into Eqs. (12) and (13) gives the final velocities of the stellar components in the original reference frame.

We find for the neutron star:

$$v_{\rm NS,x} = w\cos\vartheta\left(\frac{1}{R}+1\right) + \left(\frac{1}{R} + \frac{m_2}{1+m_{\rm shell}+m_2}\right)v \quad (51)$$

$$v_{\rm NS,y} = w\sin\vartheta\cos\varphi\left(1-\frac{1}{S}\right) + \frac{1}{S}v_{\rm im} + \frac{Q\sqrt{P}}{S}v \quad (52)$$

$$v_{\rm NS,z} = w\sin\vartheta\sin\varphi\left(\frac{1}{R}+1\right) \quad (53)$$

and for the companion star:

$$v_{\rm 2x} = \frac{-w\cos\vartheta}{m_{2f}R} - \left(\frac{1}{m_{2f}R} + \frac{1+m_{\rm shell}}{1+m_{\rm shell}+m_2}\right)v \quad (54)$$

$$v_{\rm 2y} = \frac{w\sin\vartheta\cos\varphi}{m_{2f}S} + \left(1-\frac{1}{m_{2f}S}\right)v_{\rm im} - \frac{Q\sqrt{P}}{m_{2f}S}v \quad (55)$$

$$v_{\rm 2z} = \frac{-w\sin\vartheta\sin\varphi}{m_{2f}R} \quad (56)$$

3. Results

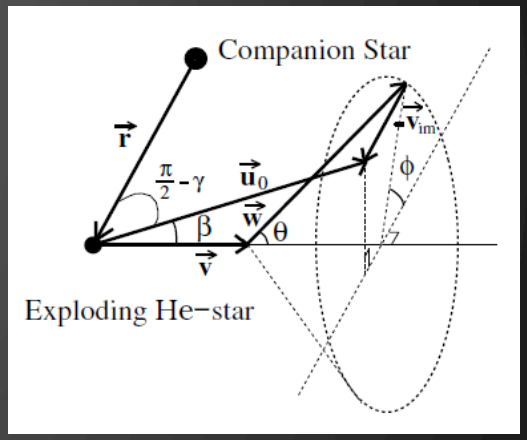
In the following subsection (Sect. 3.1) we briefly describe a few general consequences on the binary systems as a result of an asymmetric SN. In Sect. 3.2 we discuss the influence of the pre-SN orbital parameters on the velocities of neutron stars ejected from disrupted binaries. Sect. 3.3 briefly demonstrates applications to systems which either undergo a symmetric SN or remain bound after the SN event.

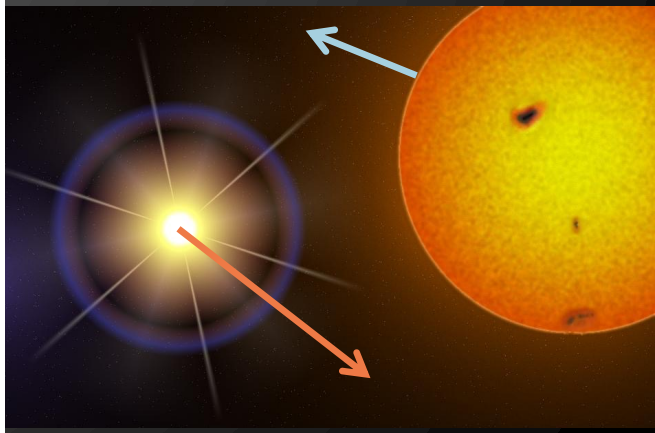
3.1. Consequences of an asymmetric SN in a binary system

3.1.1. Direction of the kick and coalescense

First of all, we illustrate how the direction of the kick, imparted to the newborn neutron star, affects the velocity (speed) of the ejected neutron star. This is demonstrated in Fig. 4 (top) as a function of the pre-SN separation, r. We have assumed a mass of $M_1 = 4.0\ M_\odot$ for the exploding He star, a neutron star mass of $M_{\rm NS} = 1.4\ M_\odot$, a companion star mass of $M_2 = 1.0\ M_\odot$

Runaway velocities of ejected stars
(Tauris & Takens, 1998)





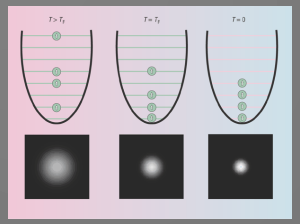
Summary

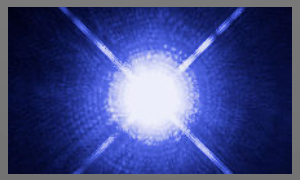
X-ray Binaries

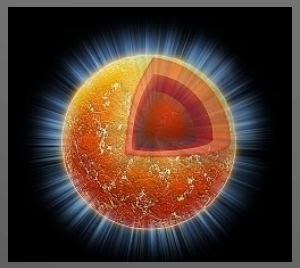
- X-ray binaries (HMXBs / LMXBs)
- Roche-lobe overflow - Cases A, B, C, and Case BB
- Stability criteria for mass transfer / stellar evolution
- Orbital angular momentum balance equation
- Common envelope and spiral-in evolution

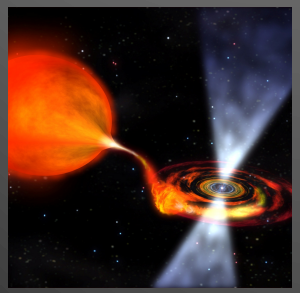
For a review: Tauris & van den Heuvel (2006)
and new textbook: Tauris & van den Heuvel (2023)

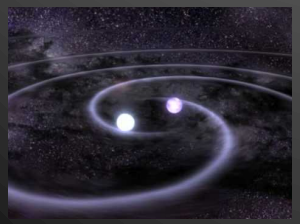
Programme





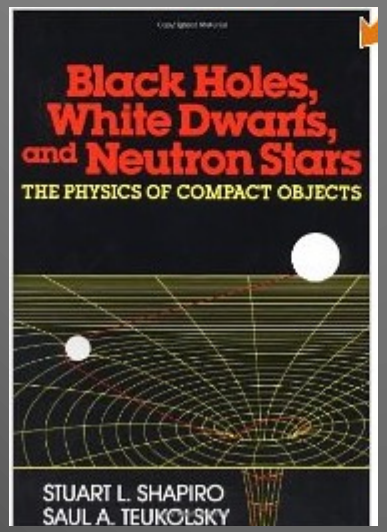






- **Introduction**
- **Degenerate Fermi Gases**
  Non-relativistic and extreme relativistic electron / (n,p,e^-) gases
- **White Dwarfs**
  Structure, cooling models, observations
- **Neutron Stars**
  Structure and equation-of-state
- **Radio Pulsars**
  Characteristics, spin evolution, magnetars, observations
- **Binary Evolution and Interactions**
  X-ray binaries, accretion, formation of millisecond pulsars, recycling
- **Black Holes**
  Observations, characteristics and spins
- **Gravitational Waves**
  Sources and detection, kilonovae
- **Exam**

Physics of Compact Objects
week 7



Shapiro & Teukolsky (1983), Wiley-Interscience

Curriculum
(Tauris & van den Heuvel (2023), Chapters 4,6,7,10,11
+ S&T Chapter 13+15)

- Next lecture: Tauris & van den Heuvel (2023), Chapters 7.3 + 14 (S&T Chapter 18). Aud.2.115.

Exercises: # 9–11, 16

- Monday Oct. 23, 10:15-12:00