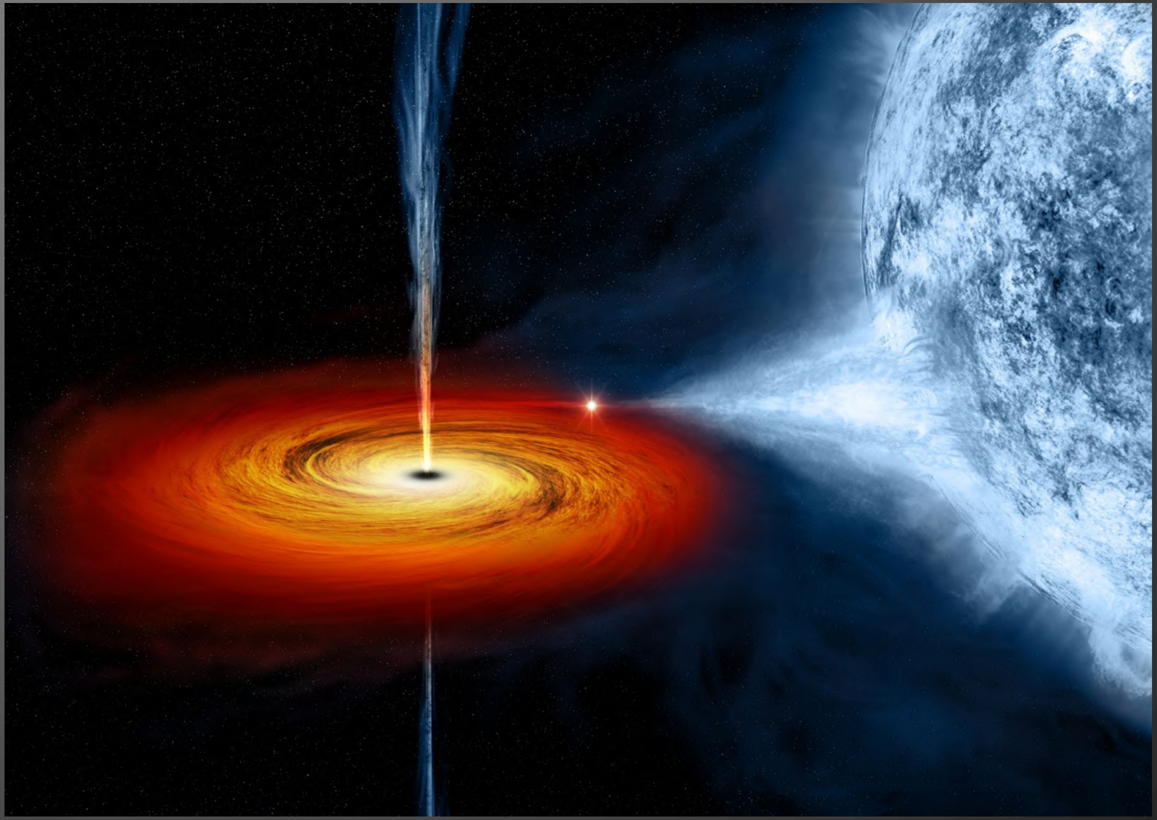
PHYSICS OF COMPACT OBJECTS

AND THEIR BINARY INTERACTIONS



AALBORG
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Thomas Tauris – Physics, Aalborg University

Last week

X-ray Binaries

- X-ray binaries (HMXBs / LMXBs)
- Roche-lobe overflow - Cases A, B, C, and Case BB
- Stability criteria for mass transfer / stellar evolution
- Orbital angular momentum balance equation
- Common envelope and spiral-in evolution

For a review: Tauris & van den Heuvel (2006)
and new textbook: Tauris & van den Heuvel (2023)

Programme

▣ ∗ **Introduction**

▣ ∗ **Degenerate Fermi Gases**

Non-relativistic and extreme relativistic electron / (n,p,e^-) gases

▣ ∗ **White Dwarfs**

Structure, cooling models, observations

▣ ∗ **Neutron Stars**

Structure and equation-of-state

▣ ∗ **Radio Pulsars**

Characteristics, spin evolution, magnetars, observations

▣ ∗ **Binary Evolution and Interactions**

X-ray binaries, accretion, formation of millisecond pulsars, recycling

▣ ∗ **Black Holes**

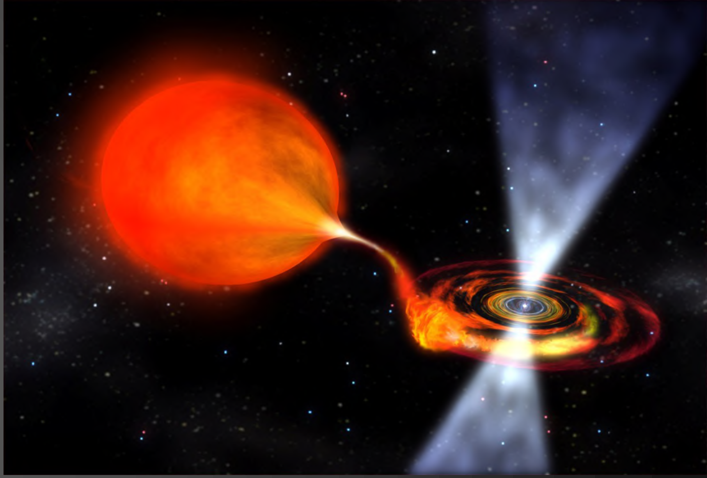
Observations, characteristics and spins

▣ ∗ **Gravitational Waves**

Sources and detection, kilonovae

▣ ∗ **Exam**

Recyling MSPs
– accretion physics

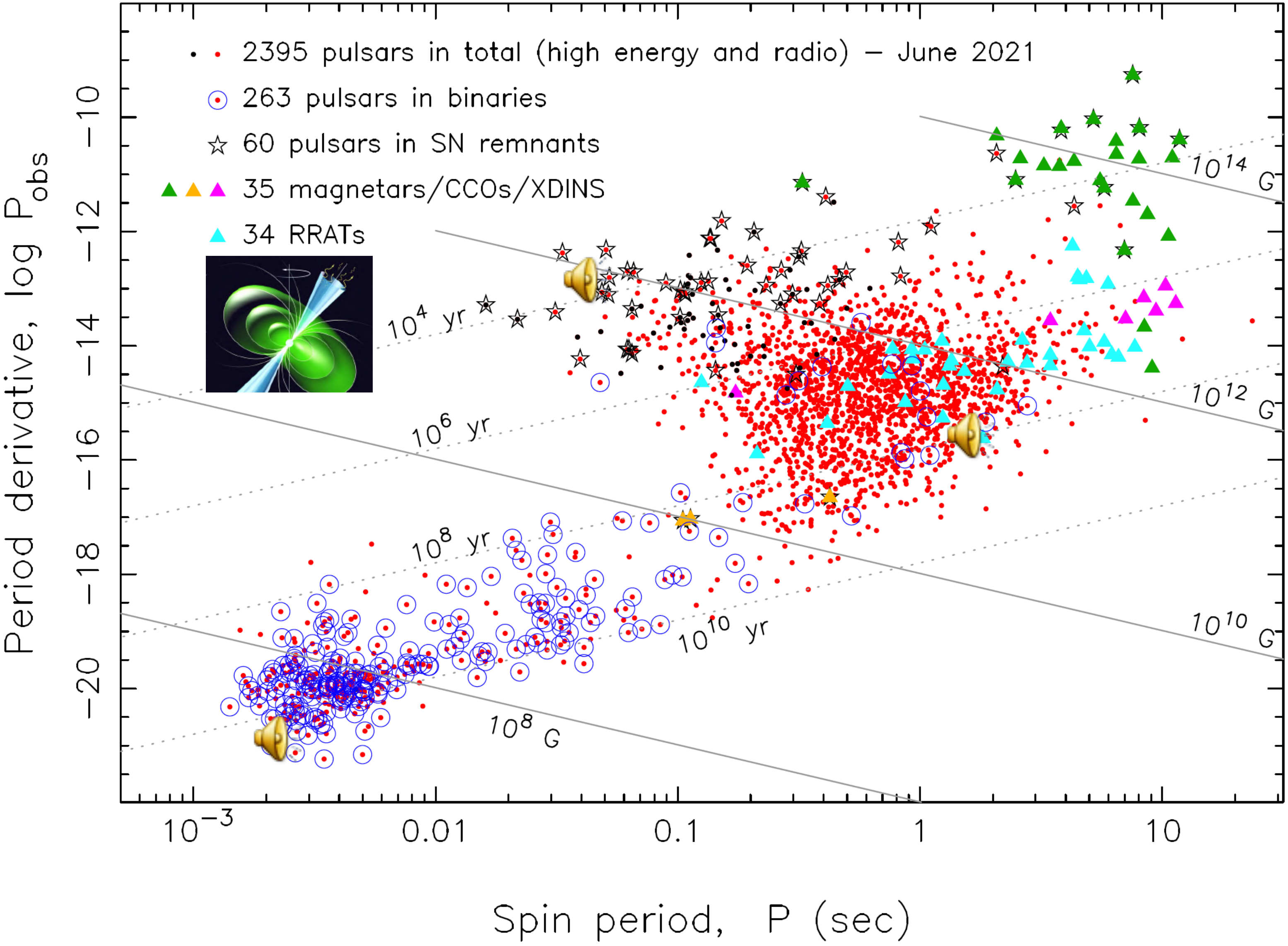


- Detailed LMXB evolution
  - dependence on P_{orb} and M_2
  - relation between M_{WD} and P_{orb} for binary pulsars
  - mass-transfer rate; final neutron star mass
  - equilibrium spin period and spin-up line in P-P_{dot} diagram

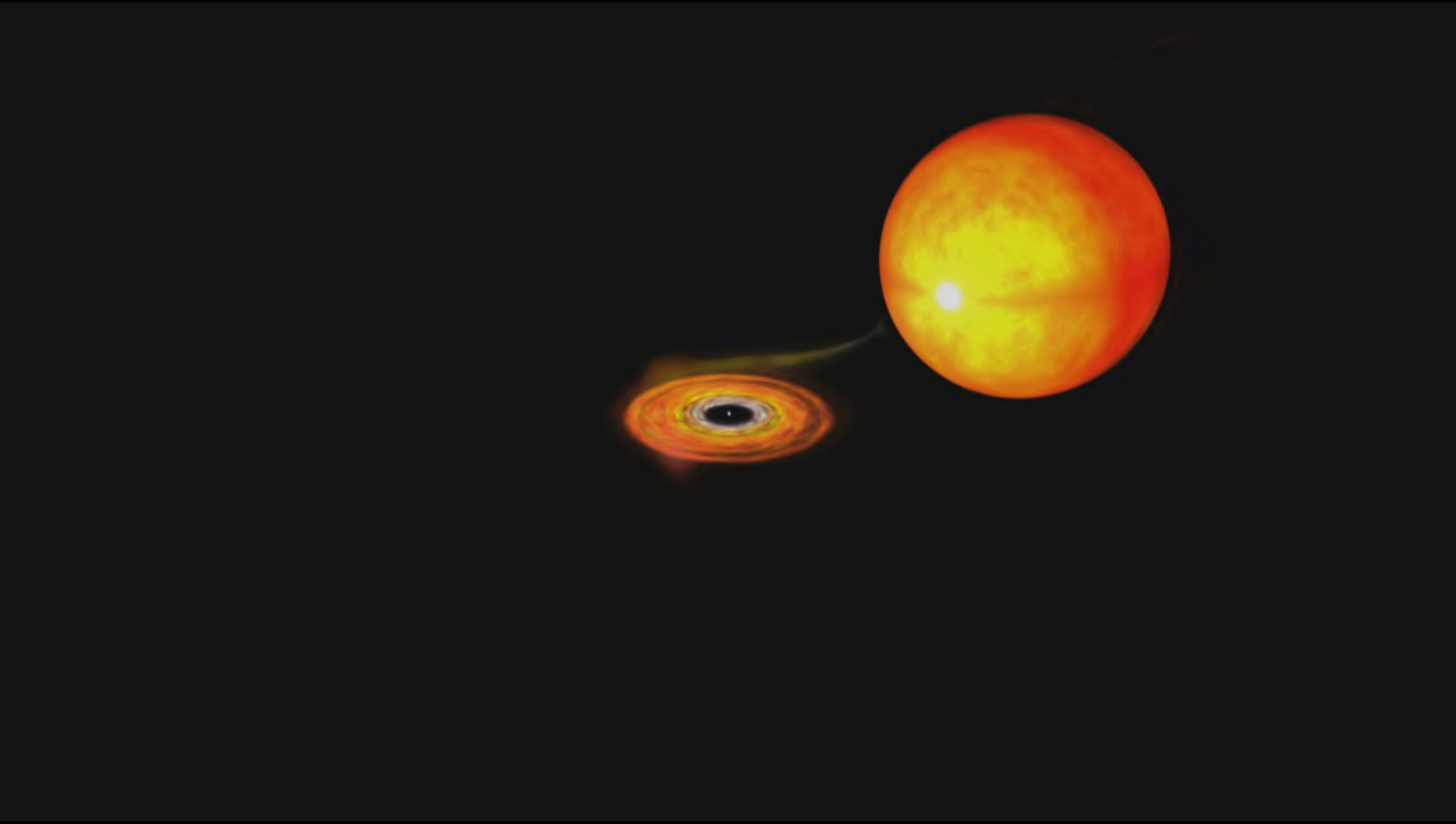
- Accretion physics
  - Four phases of accretion
  - Accretion disks
  - Accretion-induced magnetic field decay

Millisecond pulsars

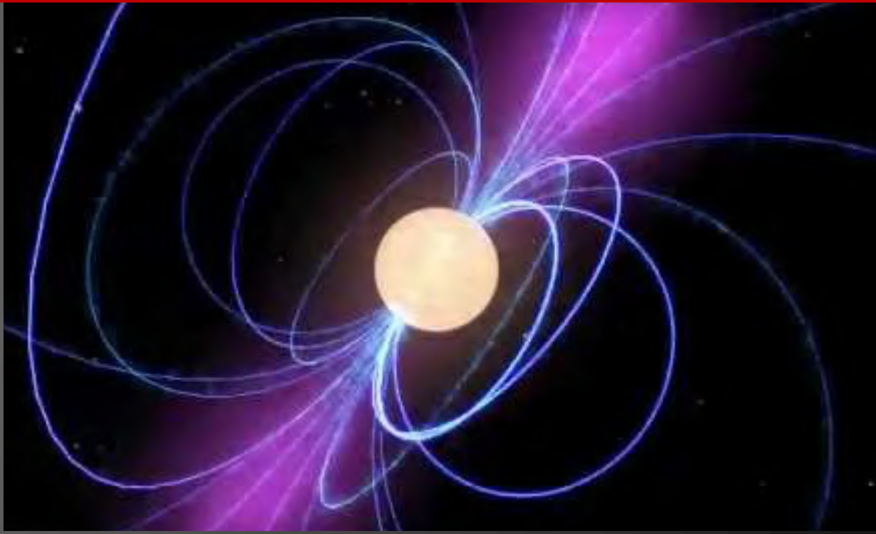
Tauris & van den Heuvel (2023)



Millisecond Pulsars - a binary formation scenario



Millisecond pulsars - Key questions on their origin



MSPs

- How fast can they spin?
- Why are their B-field strenghts weak?
- How much mass do they need to accrete?
- What are their orbital periods?
- How old are they?
- Where are they located in our Galaxy (+ kinematics)?
- What is the nature of their companion stars?

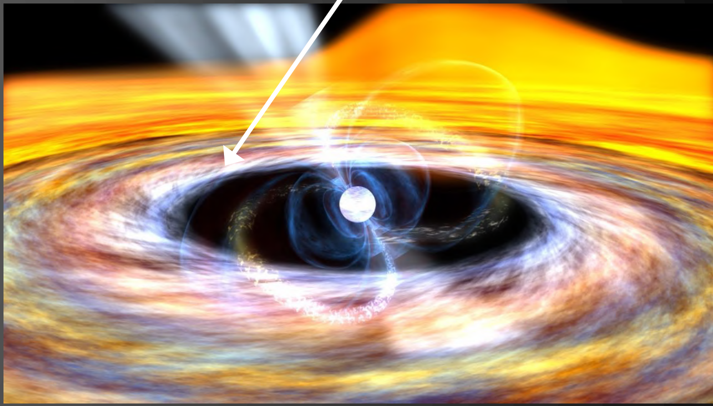
➡ How do MSPs form?

Millisecond pulsars - a binary formation scenario

Why?

- Rapid spin: $P < 50\ ms$
- Small period derivative: $\dot{P} < 10^{-17}\ s\,s^{-1}$

$$\dot{J} = \frac{d}{dt}|\vec{r} \times \vec{p}\,|$$



Solution:

- Accretion of mass

$$N = \dot{J}_* \equiv \frac{d}{dt}(I\Omega_*) = \dot{M}_*\sqrt{GM_* r_A}\ \xi$$

Lamb, Pethick & Pines (1973)
Ghosh & Lamb (1979, 1992)

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v} \times \vec{B}) - \frac{c^2}{4\pi}\nabla \times \left(\frac{1}{\sigma} \times \nabla \times \vec{B}\right)$$

Geppert & Urpin (1994); Konar & Bhattacharya (1997)

$$B = \sqrt{\frac{3c^3 I_{NS}}{8\pi^2 R_{NS}^6} P\dot{P}}$$

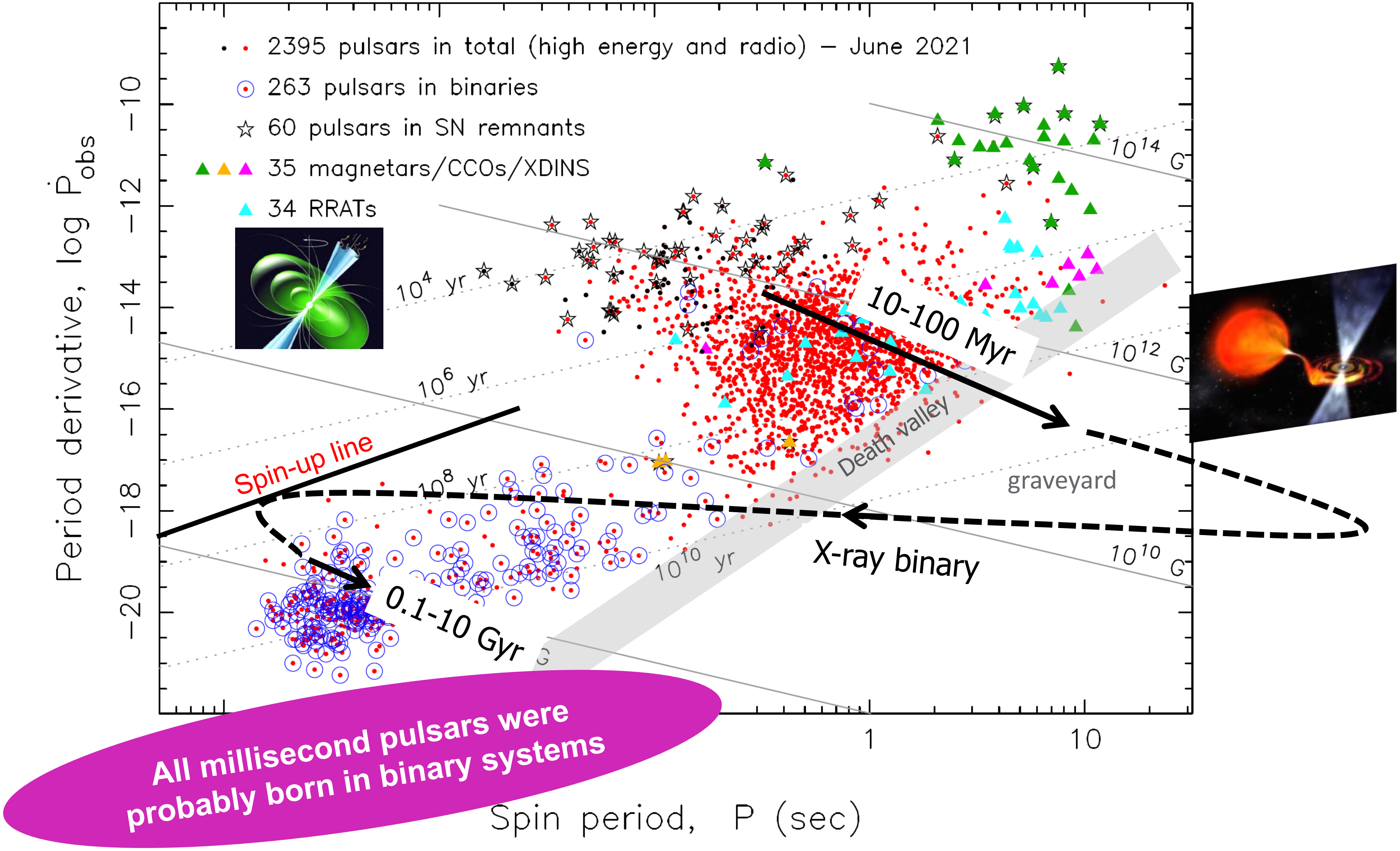
Magnetic-dipole model

Accretion-induced B-field decay
– Ohmic dissipation/diffusion
– Flux tube expulsion via spin-down
– B-field burial (screening)

Thomas Tauris

Recycling pulsars - A detour in the P-Pdot diagram

Tauris & van den Heuvel (2023)

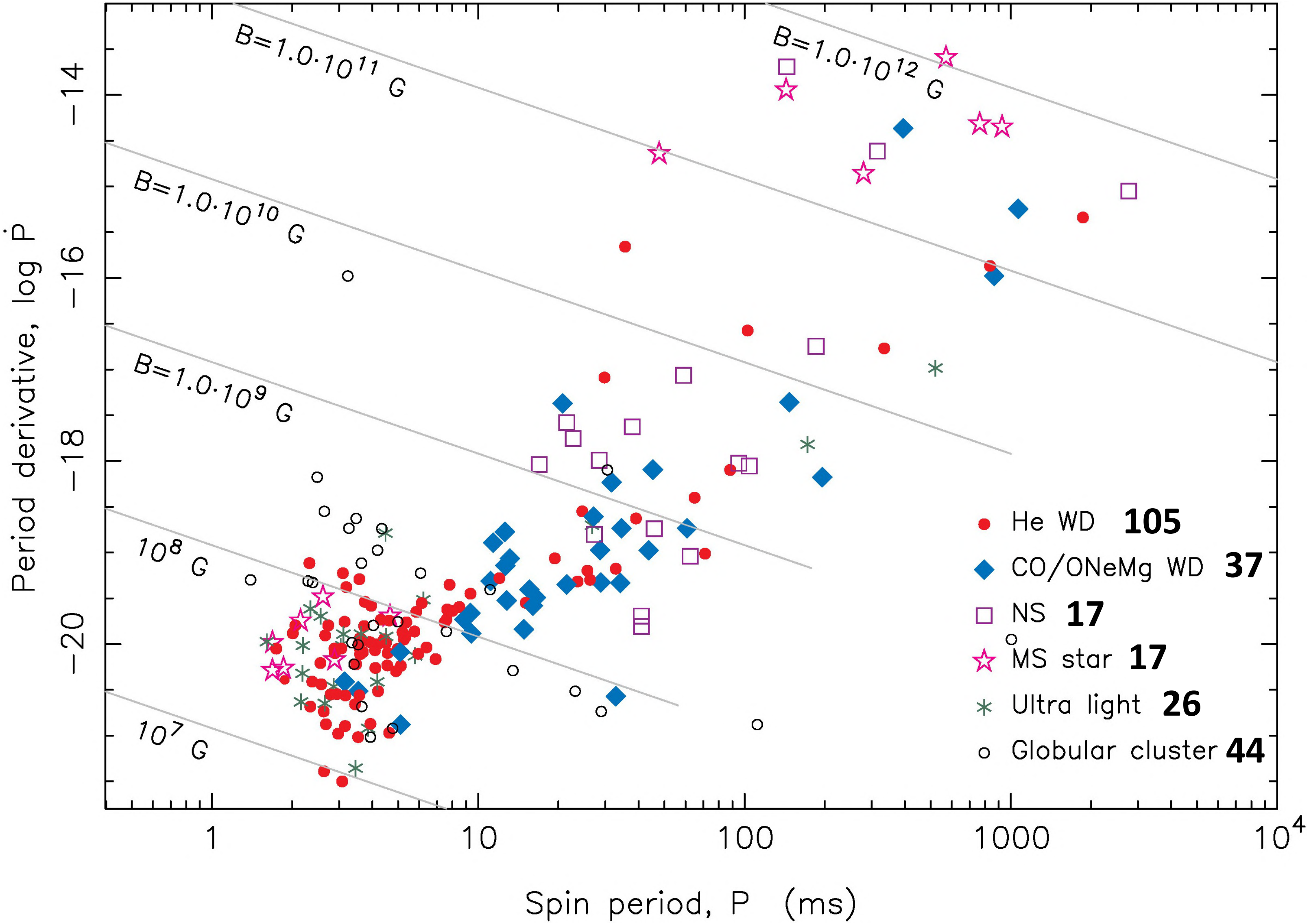


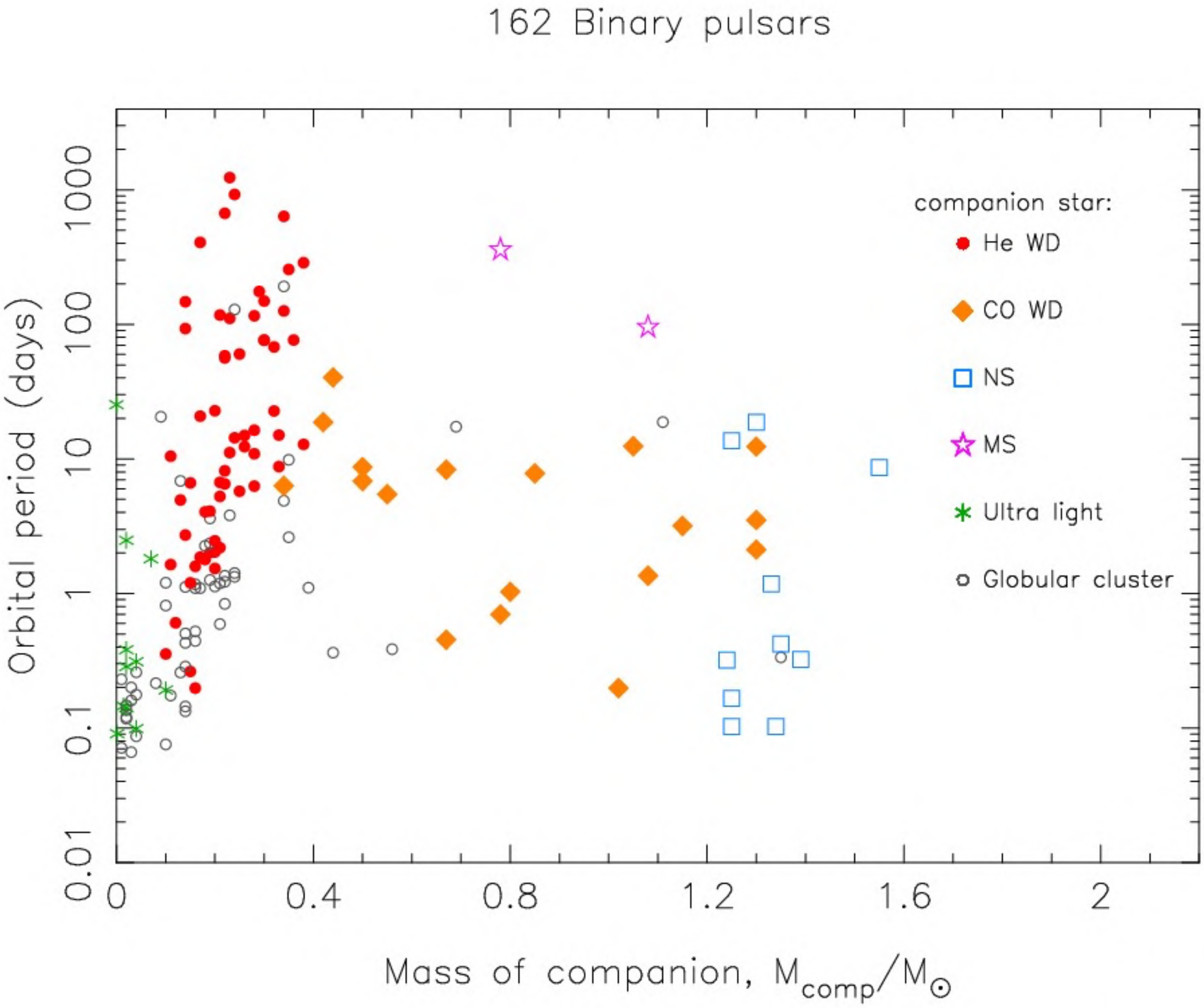
All millisecond pulsars were probably born in binary systems

Pulsar companion stars

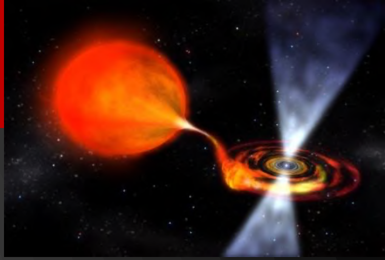
245 binary pulsars in the Galactic disk with measured $\dot{P}$

Tauris & van den Heuvel (2023)




162 Binary pulsars
Orbital period (days)
0.01
0.1
1
10
100
1000
0
0.4
0.8
1.2
1.6
2
Mass of companion, $M_{comp}/M_{\odot}$
companion star:
● He WD
◆ CO WD
□ NS
☆ MS
✱ Ultra light
○ Globular cluster

LMXB bifurcation period



$P_{orb} < P_{bif}$: Converging

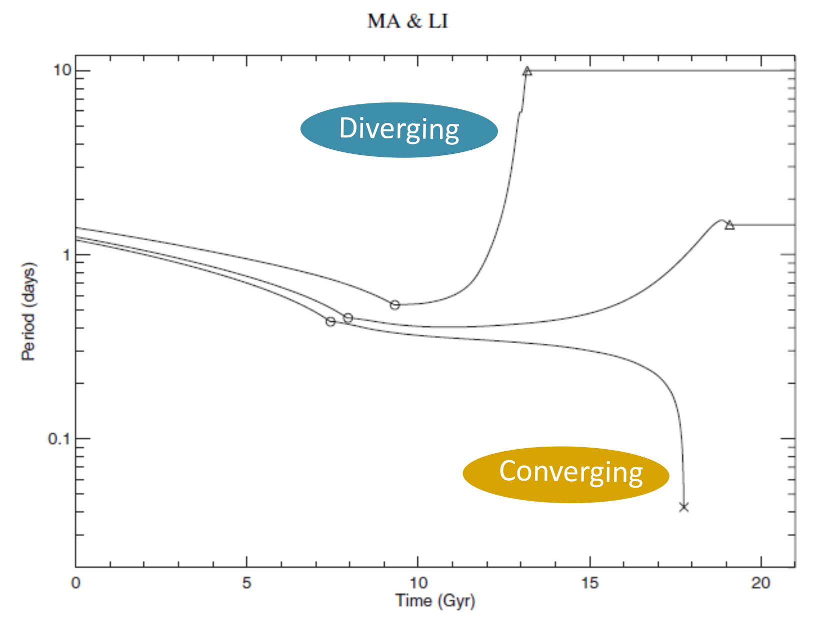
→ LMXB **shorten** their orbital period

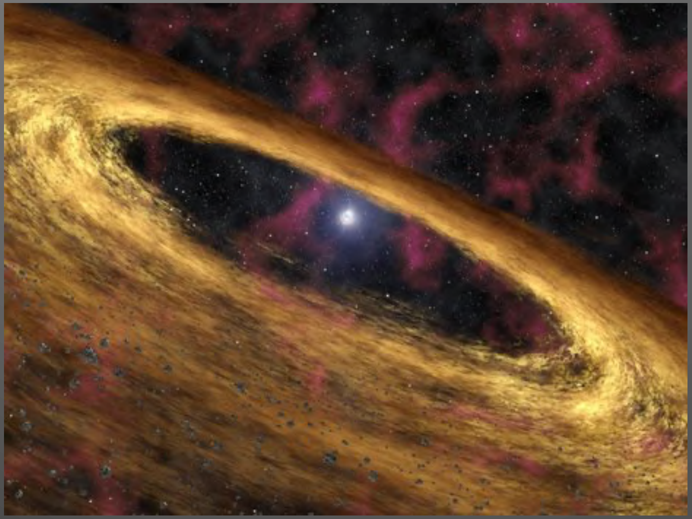
Donor star still on main sequence

RLO driven by loss of J_{orb} (MB, GWs)

Tutukov et al. (1985)
Pylyser & Savonije (1988, 1989)
Ma & Li (2009)
Istrate et al. (2014)
Chen et al. (2021)

$P_{bif} \approx 1\ day$





"Black widow" millisecond pulsars: EoS?

$P_{orb} < 10\ hrs \qquad M_{comp.} < 0.1\ M_{\odot}$

Evaporation → single millisecond pulsars ?

Roberts (2012)
Breton et al. (2013)
Chen et al. (2013)
Conrad-Burton et al. (2023)
Koljonen & Linares (2023)

Single millisecond pulsars

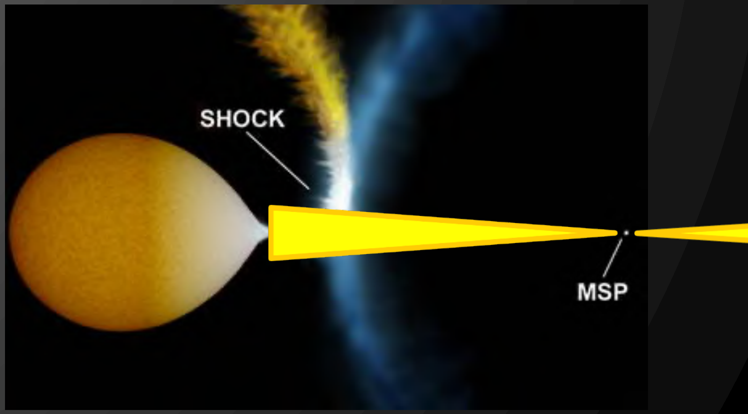
All single millisecond pulsars are likely born in a binary system.

Once a recycled millisecond pulsar turns on its emission of ultra-relativistic particles, it is often able to completely evaporate its companion and thus end up as an isolated millisecond pulsar.

Observational evidence:

✓ eclipsing MSPs with $0.02\ M_\odot$ companions

✓ the ”planetary pulsar”, PSR 1257+12

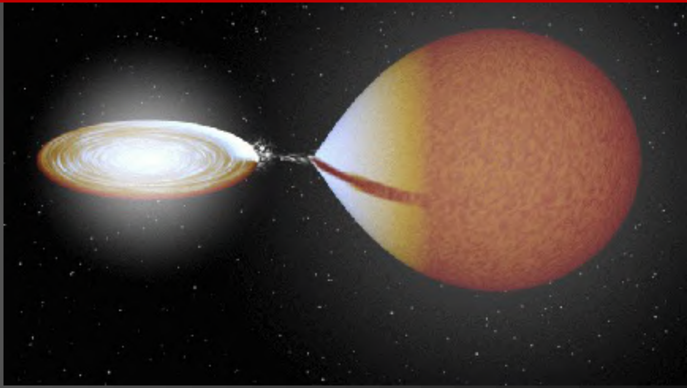


power needed for evaporation = incoming irradiation power from pulsar

$$\frac{1}{2}\dot{M}_2 v_{esc}^2 = f\,\dot{E}_{psr}\left(\pi R_2^2/4\pi a^2\right) \qquad \tau \sim \frac{M_2}{\dot{M}_2}$$

evaporation timescale

LMXB diverging systems



$P_{orb} > P_{bif}$: Diverging

→ <u>LMXB **widen** their orbital period</u>

Donor star is a (sub)giant

RLO driven by nuclear expansion

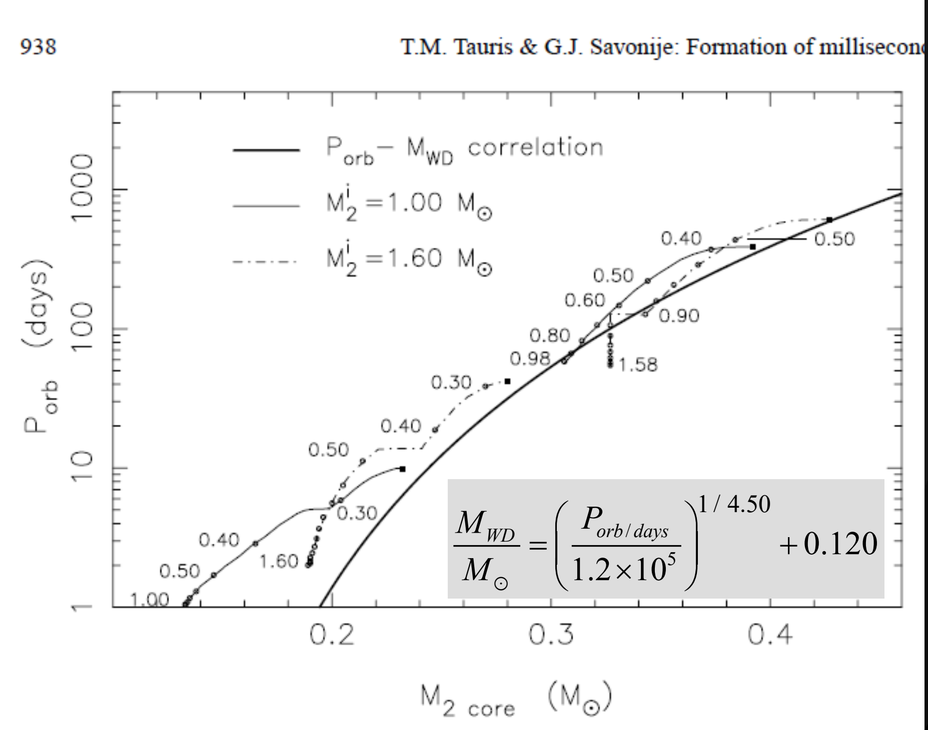
Formation of BMSPs with He-WD:

$$P_{orb} > 1\ day$$
$$0.18 < M_{WD} < 0.46\ M_\odot$$

Unique relation between P_{orb} and M_{WD}

Savonije (1987)
Joss, Rappaport & Lewis (1987)
Rappaport et al. (1995)
Tauris & Savonije (1999)
Istrate et al. (2016)

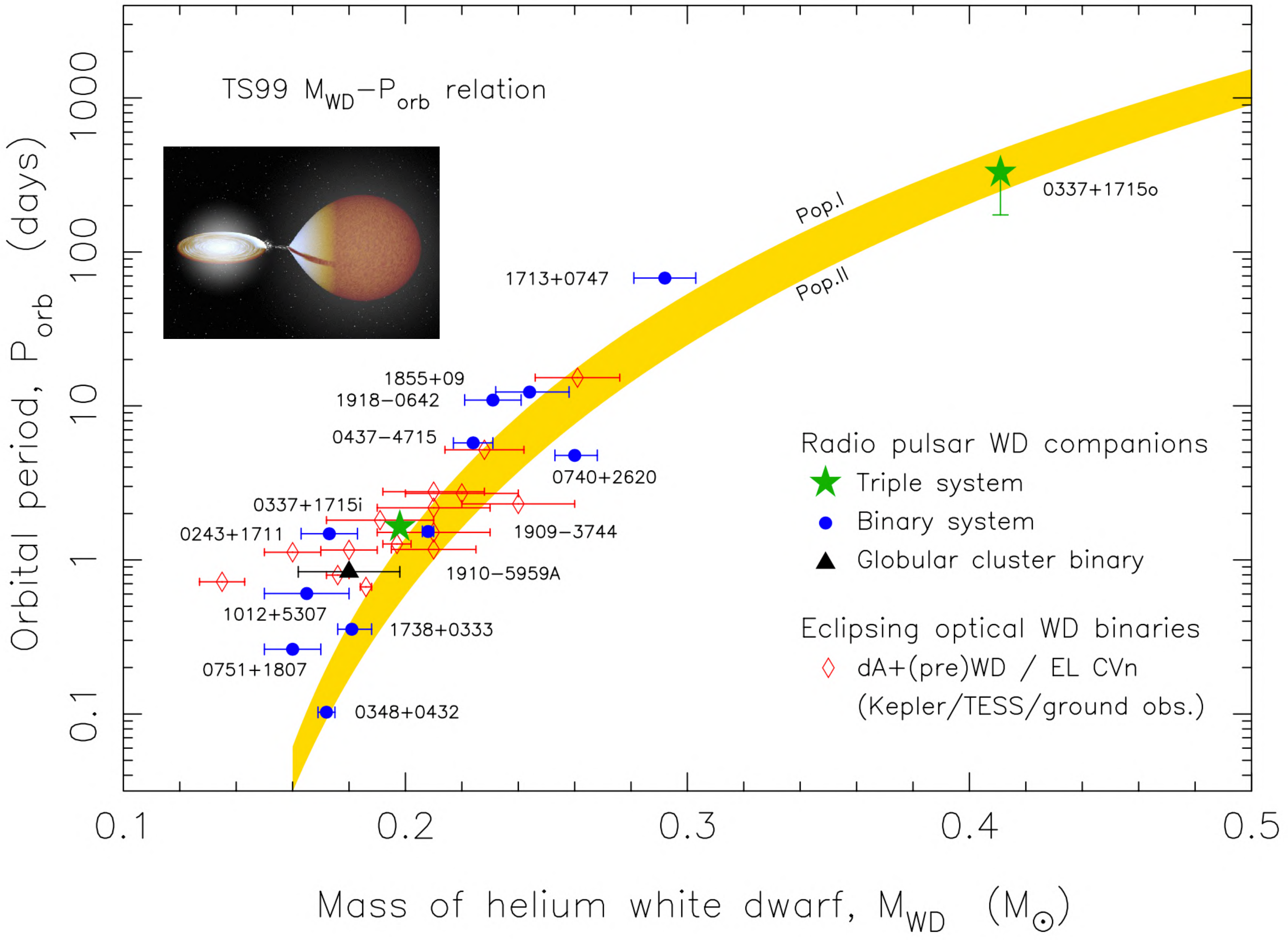
match observations well



938 T.M. Tauris & G.J. Savonije: Formation of millisecon

$$\frac{M_{WD}}{M_\odot} = \left(\frac{P_{orb/days}}{1.2\times10^5}\right)^{1/4.50} + 0.120$$

P_{orb} – M_{WD} correlation for He-WDs



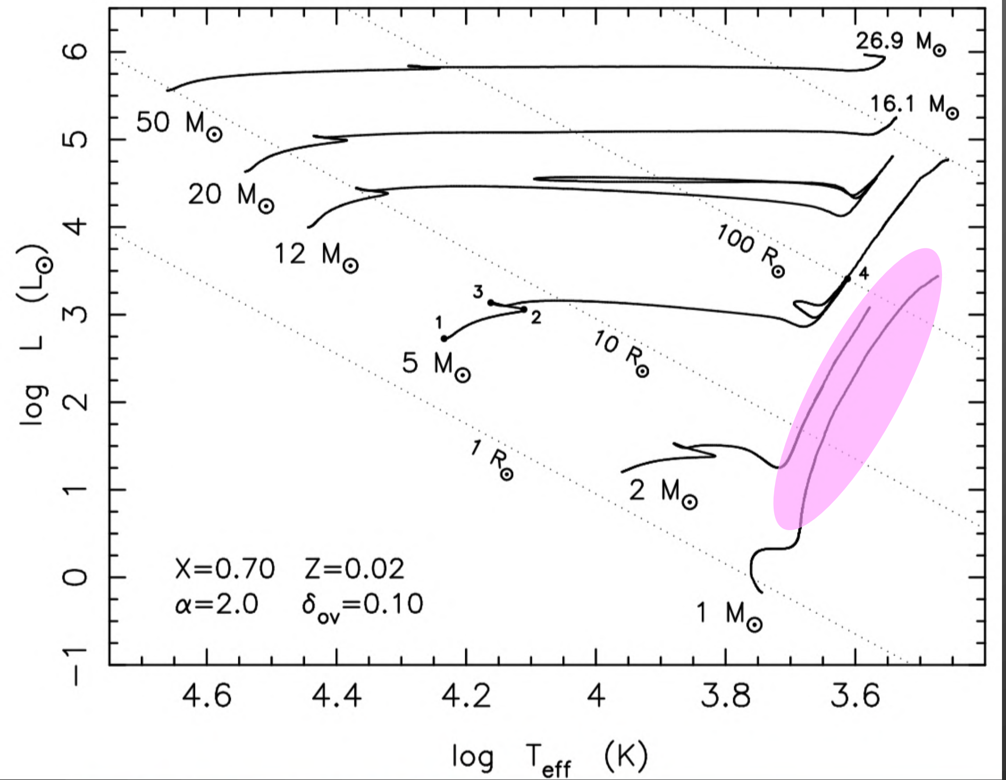
Tauris & van den Heuvel (2023)

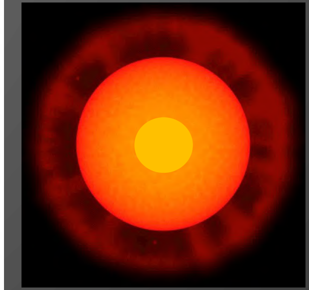
P_{orb} – M_{WD} correlation for He-WDs

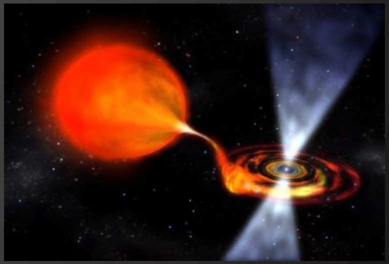
- On the red giant branch (hydrogen shell burning) the growth of the degenerate He core mass is directly related to the luminosity of the star
- Temperature is almost constant on the Hyashi track $\Rightarrow$ $L \propto R^2$

$$L = 4\pi R^2 \sigma T_{eff}^4$$

- Hence there is a relation between M_{core} and R (Thomas 1967) independent of M_{env}
- The donor star fills its Roche-lobe during the mass transfer $\Rightarrow$ R is correlated with P_{orb}







correlation between (P_{orb} , M_{WD})

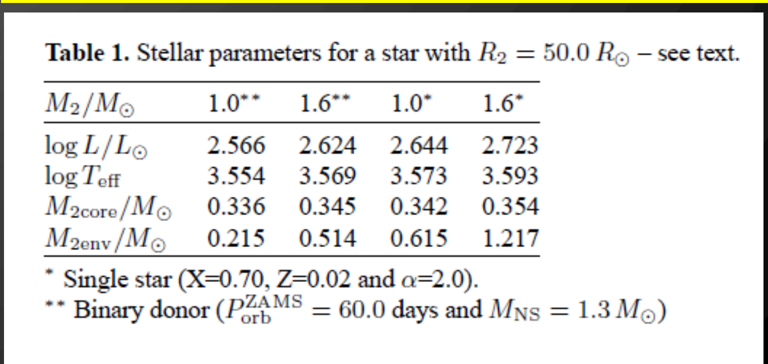


Table 1. Stellar parameters for a star with $R_2 = 50.0\,R_\odot$ – see text.

$M_2/M_\odot$	1.0^{**}	1.6^{**}	1.0^{*}	1.6^{*}
$\log L/L_\odot$	2.566	2.624	2.644	2.723
$\log T_{\rm eff}$	3.554	3.569	3.573	3.593
$M_{\rm 2core}/M_\odot$	0.336	0.345	0.342	0.354
$M_{\rm 2env}/M_\odot$	0.215	0.514	0.615	1.217

* Single star (X=0.70, Z=0.02 and α=2.0).
** Binary donor ($P_{\rm orb}^{\rm ZAMS} = 60.0$ days and $M_{\rm NS} = 1.3\,M_\odot$)

A perfect circle

Accretion efficiency, $\varepsilon < 1/3$

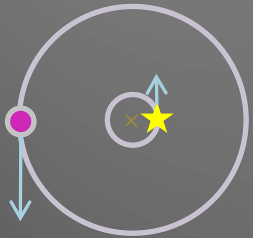
PSR J1738+0333

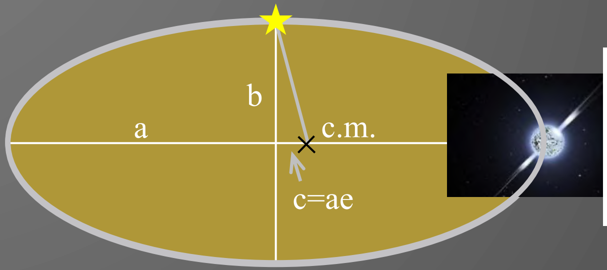
$$P = 5.85\ ms, \quad P_{orb} = 8.5\ hr, \quad M_{WD} = 0.181 \pm 0.006\ M_\odot, \quad M_{NS} = 1.47 \pm 0.06\ M_\odot$$

$$e = 3.5 \times 10^{-7}$$

Antoniadis et al. (2012)

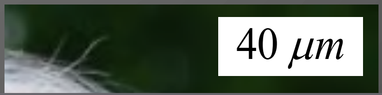
assume an ellipse in flat spacetime:





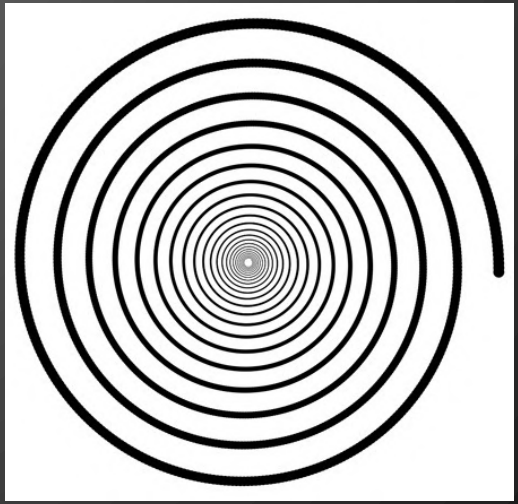
$$b^2 + c^2 \approx a^2 \quad \wedge \quad c = ae \quad \Leftrightarrow \quad b^2 = a^2(1-e^2)$$

$$\Leftrightarrow \quad |b-a| = \left| a\left(\sqrt{1-e^2} - 1\right) \right| \approx \tfrac{1}{2} a e^2 \approx 6.3\ \mu m$$

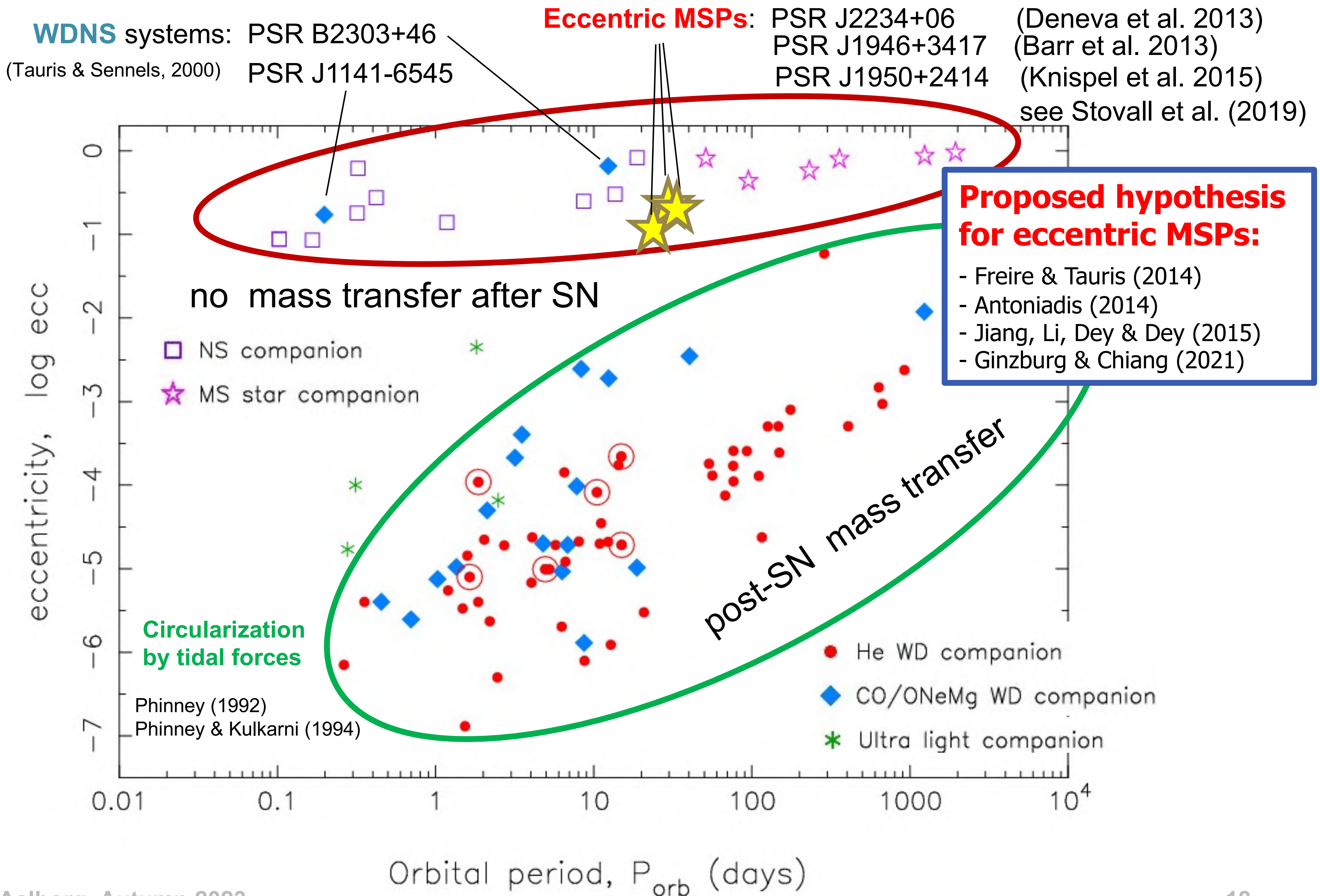


Note, orbital decay due to grav. wave rad.

$$\frac{\dot a}{a} = -\frac{64}{5}\frac{G^3}{c^5}\frac{M_1 M_2 M}{a^4} \quad \Rightarrow \quad \Delta a = 30\ \mu m \quad \text{per orbit}$$



Eccentricities



WDNS systems: PSR B2303+46

(Tauris & Sennels, 2000) PSR J1141-6545

Eccentric MSPs: PSR J2234+06 (Deneva et al. 2013)
PSR J1946+3417 (Barr et al. 2013)
PSR J1950+2414 (Knispel et al. 2015)
see Stovall et al. (2019)

no mass transfer after SN

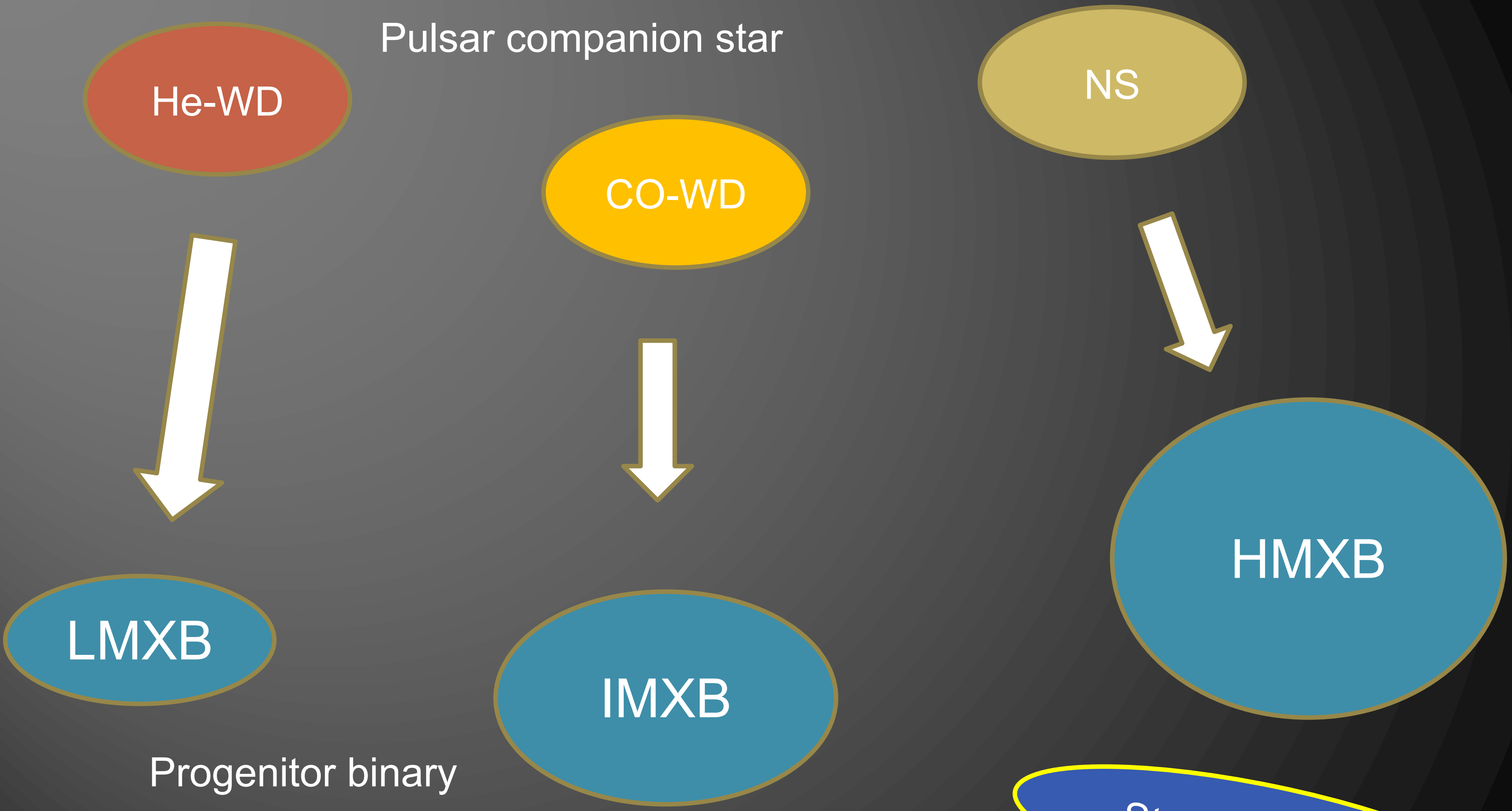
post-SN mass transfer

Circularization by tidal forces

Phinney (1992)
Phinney & Kulkarni (1994)

Proposed hypothesis for eccentric MSPs:

- Freire & Tauris (2014)
- Antoniadis (2014)
- Jiang, Li, Dey & Dey (2015)
- Ginzburg & Chiang (2021)



Pulsar companion star

He-WD

CO-WD

NS

LMXB

IMXB

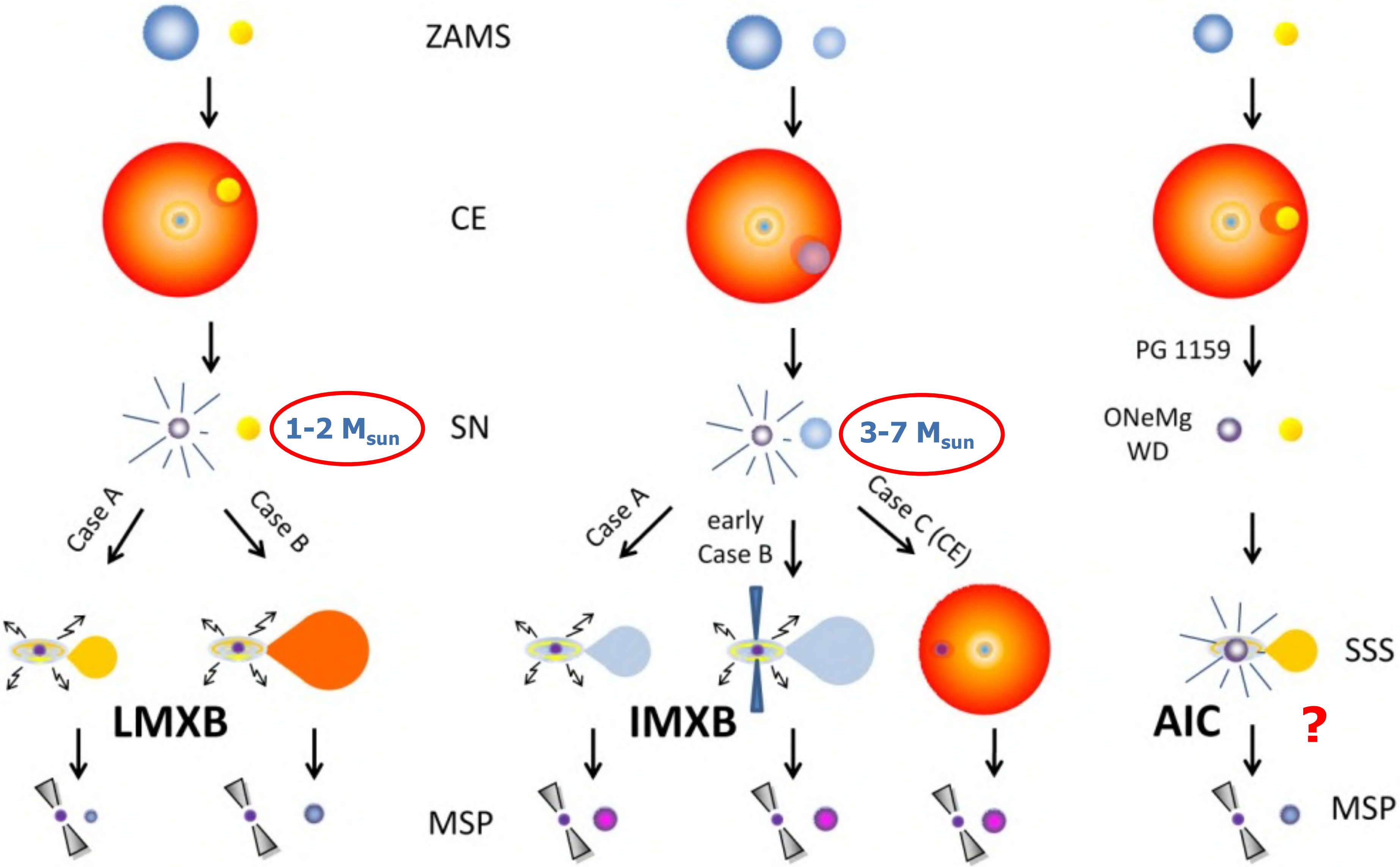
HMXB

Progenitor binary

Stellar evolution

Initial orbital period is very important!

Tauris (2011)



ZAMS

CE

PG 1159

SN

1-2 M_{sun}

3-7 M_{sun}

ONeMg WD

Case A

Case B

Case A

early Case B

Case C (CE)

SSS

LMXB

IMXB

AIC

?

MSP

MSP

low-mass He WD
sub-stellar dwarf
single MSP

redbacks
black widows
planets

He WD

CO WD
(He WD)

CO WD

CO WD
ONeMg WD

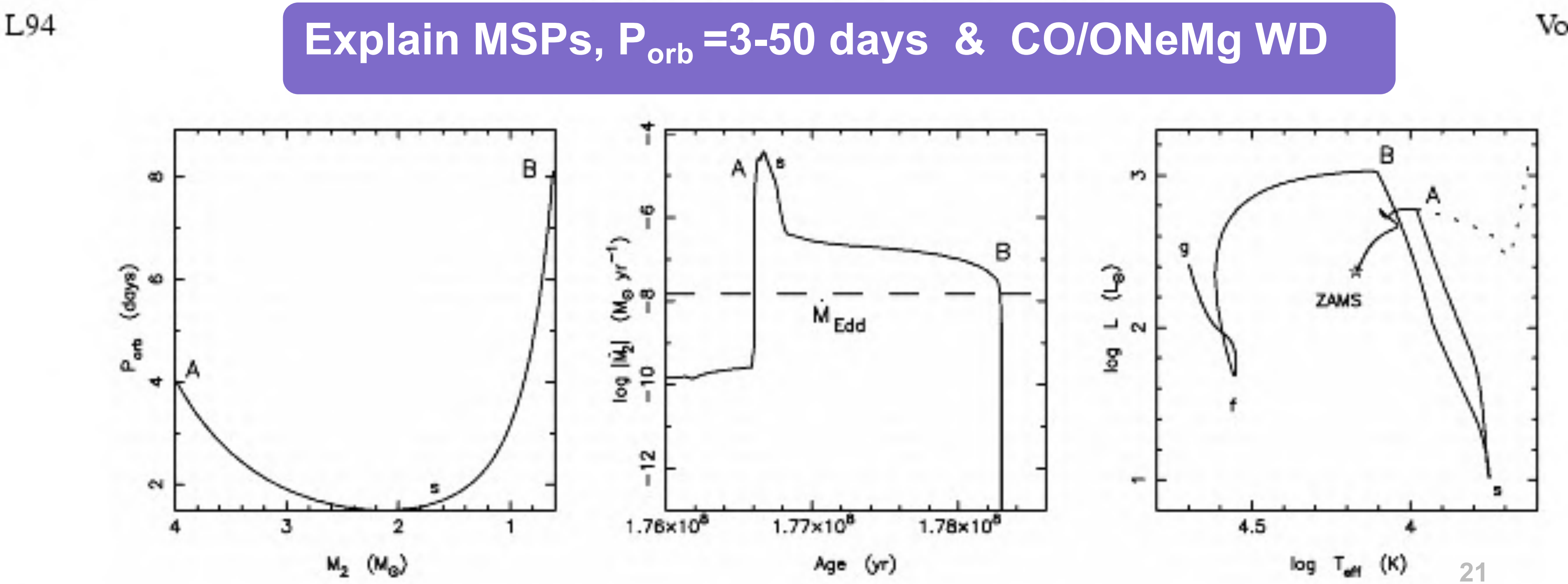
He WD
(CO WD)

IMXB Early Case B RLO → MSPs with CO-WD

Alternative to CE-phase:
- thermal timescale mass transfer
- isotropic re-emission model

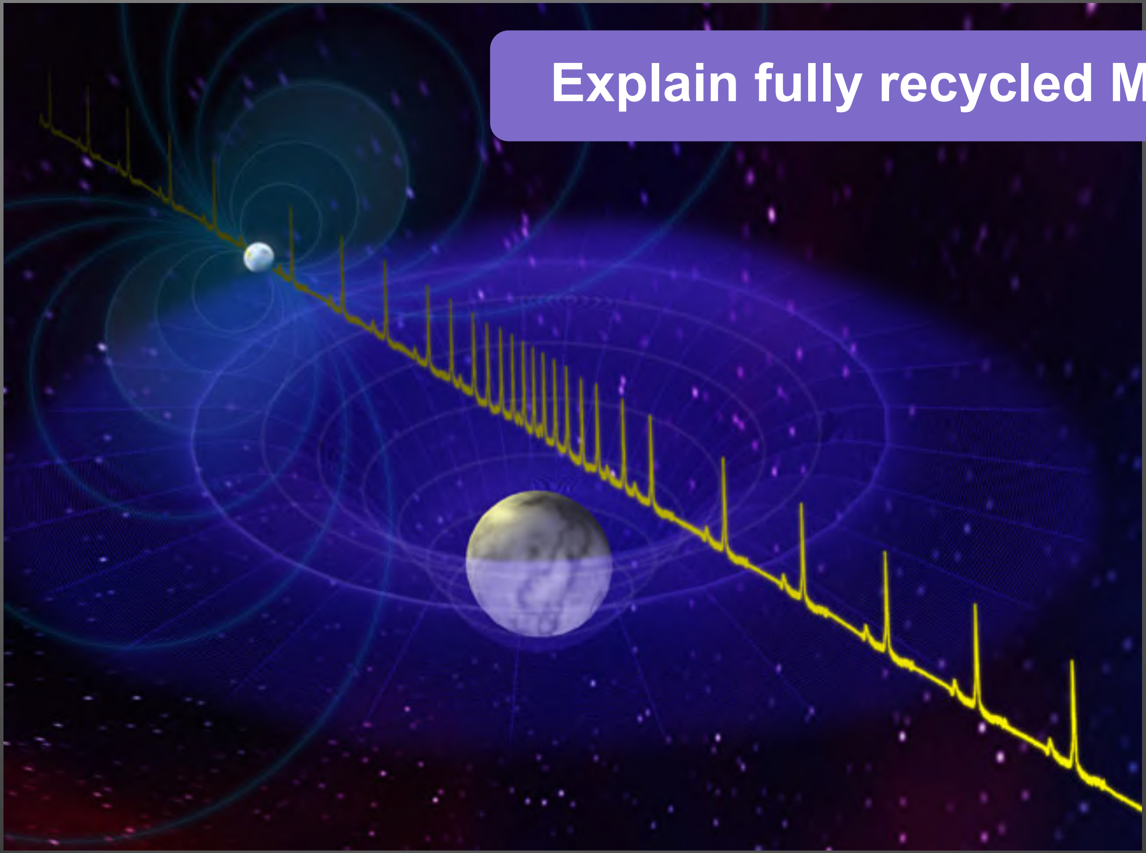
Tauris, van den Heuvel & Savonije (2000), ApJ Lett. 530, 93

Explain MSPs, P_{orb} =3-50 days & CO/ONeMg WD



IMXB Case A RLO → MSPs with CO-WD

Podsiadlowski, Rappaport & Pfahl (2002)
Tauris, Langer & Kramer (2011)



Explain fully recycled MSPs & CO/ONeMg WD

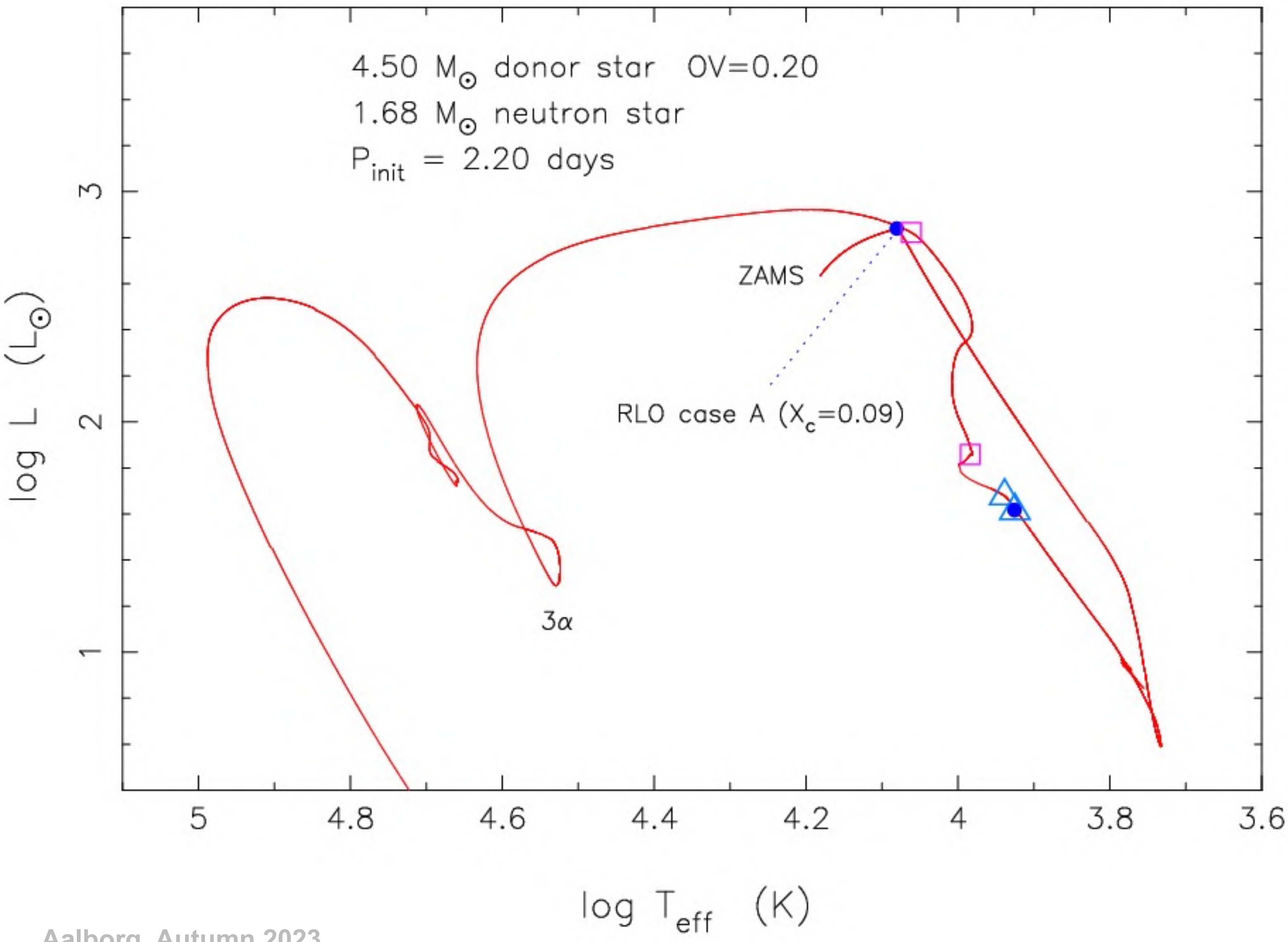
PSR J1614-2230

Demorest et al. (2010)

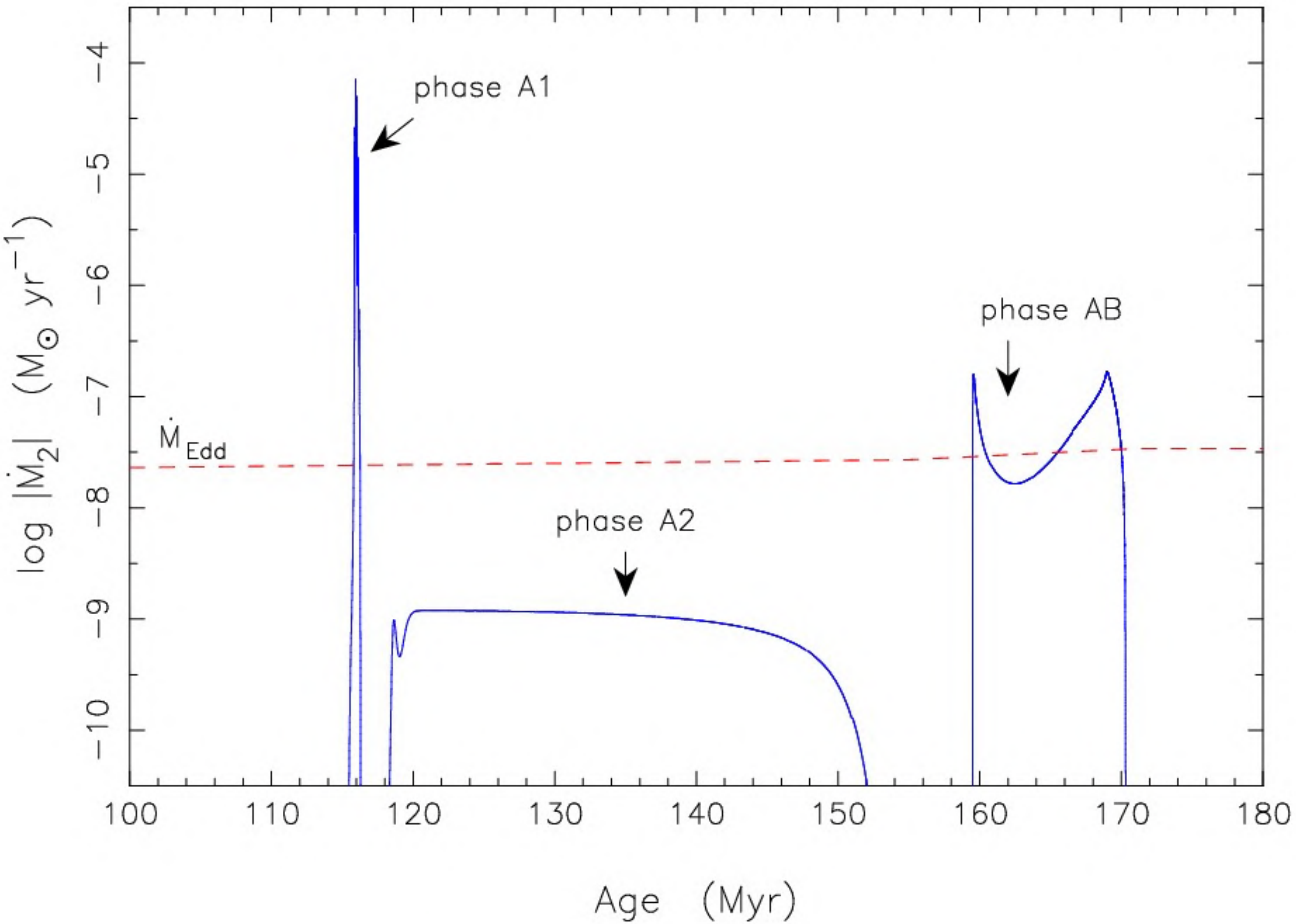
Pulsar mass:	1.97±0.04 M_o
WD mass:	0.500±0.006 M_o
Orbital period:	8.69 days
Pulsar spin period:	3.15 ms

Was this pulsar born massive?

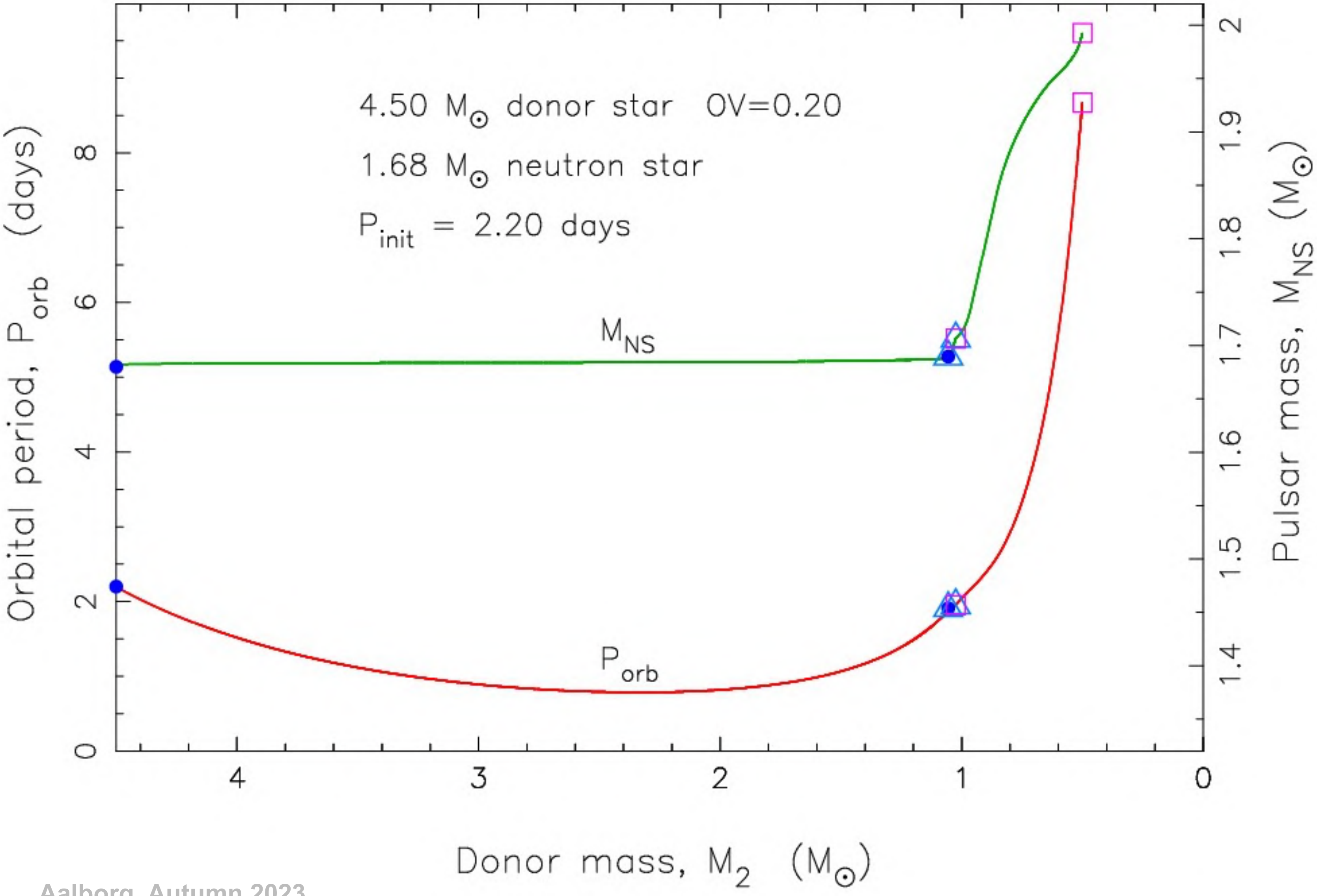
IMXB case A


4.50 $M_\odot$ donor star OV=0.20
1.68 $M_\odot$ neutron star
P_{init} = 2.20 days
ZAMS
RLO case A (X_c=0.09)
3α
log L ($L_\odot$)
1
2
3
log T_{eff} (K)
5
4.8
4.6
4.4
4.2
4
3.8
3.6

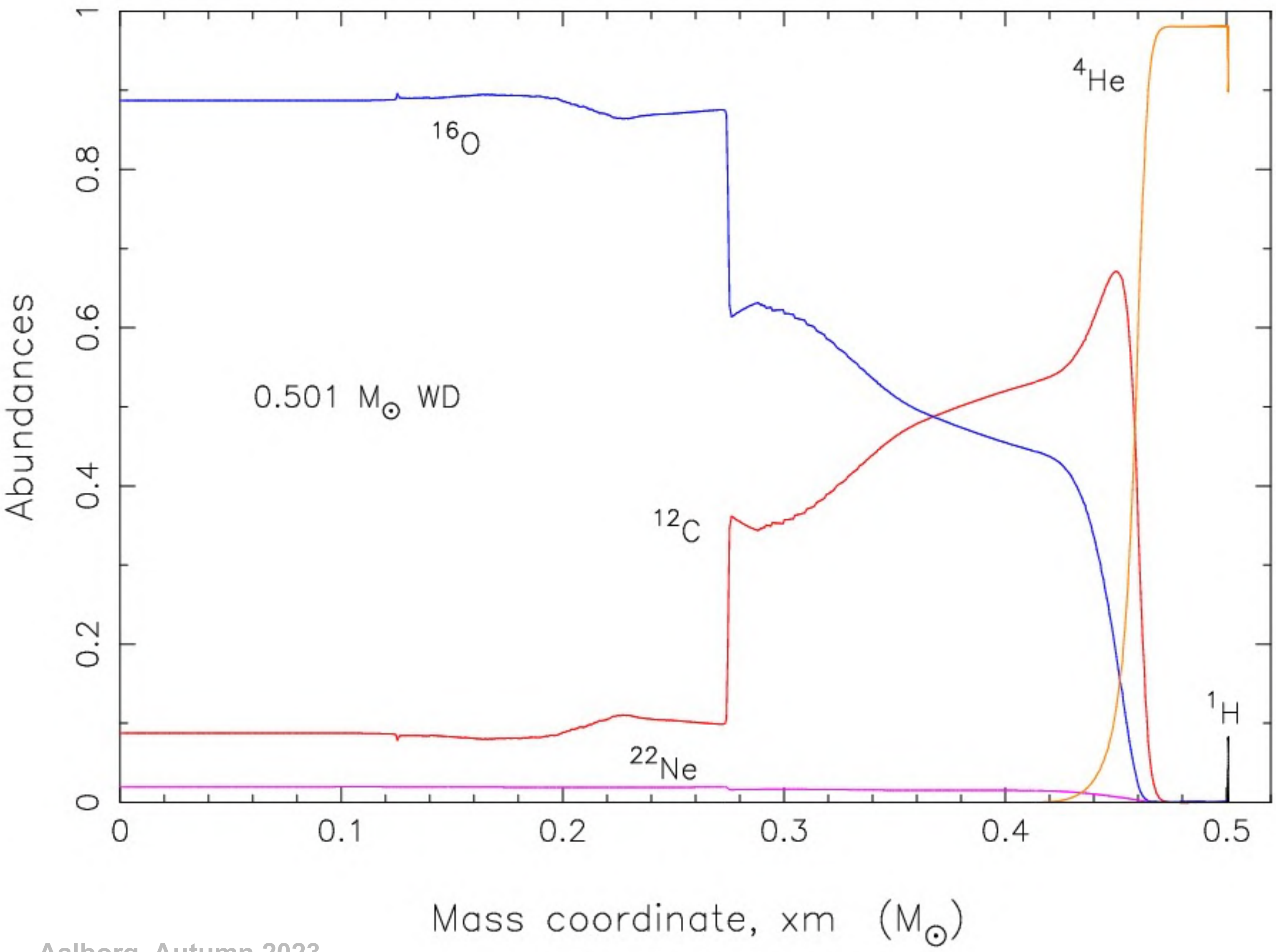
IMXB case A: thermal + nuclear timescale mass transfer



The pulsar was born with a mass of 1.7±0.1 M_{sun}



Chemical structure of the CO WD companion

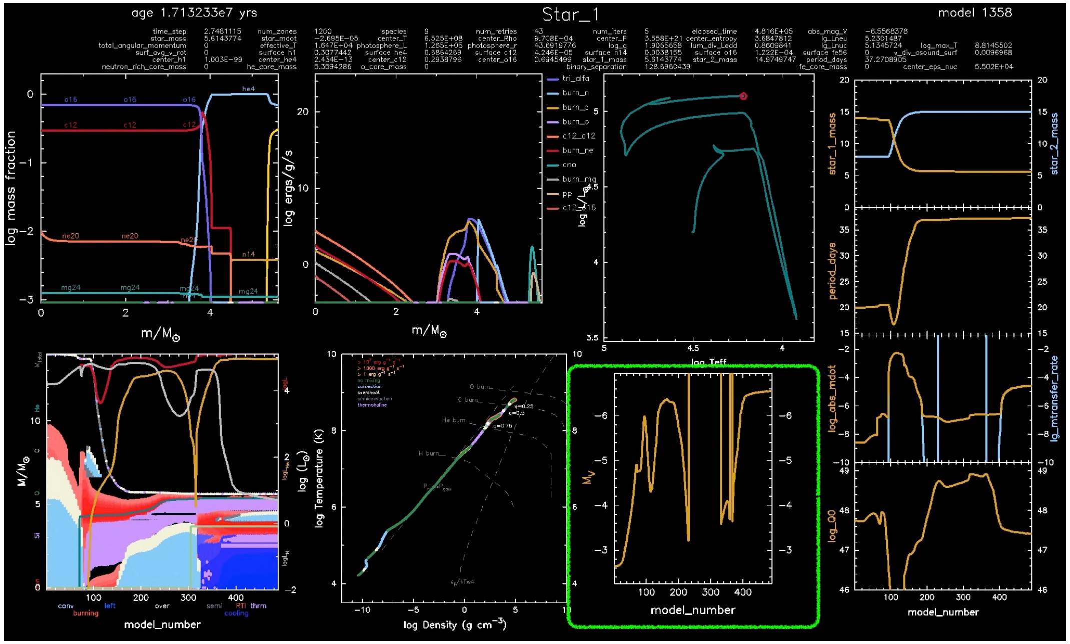


For calculating binary star evolution, I recommend using the MESA code

(e.g. for a 12 months Master's project)

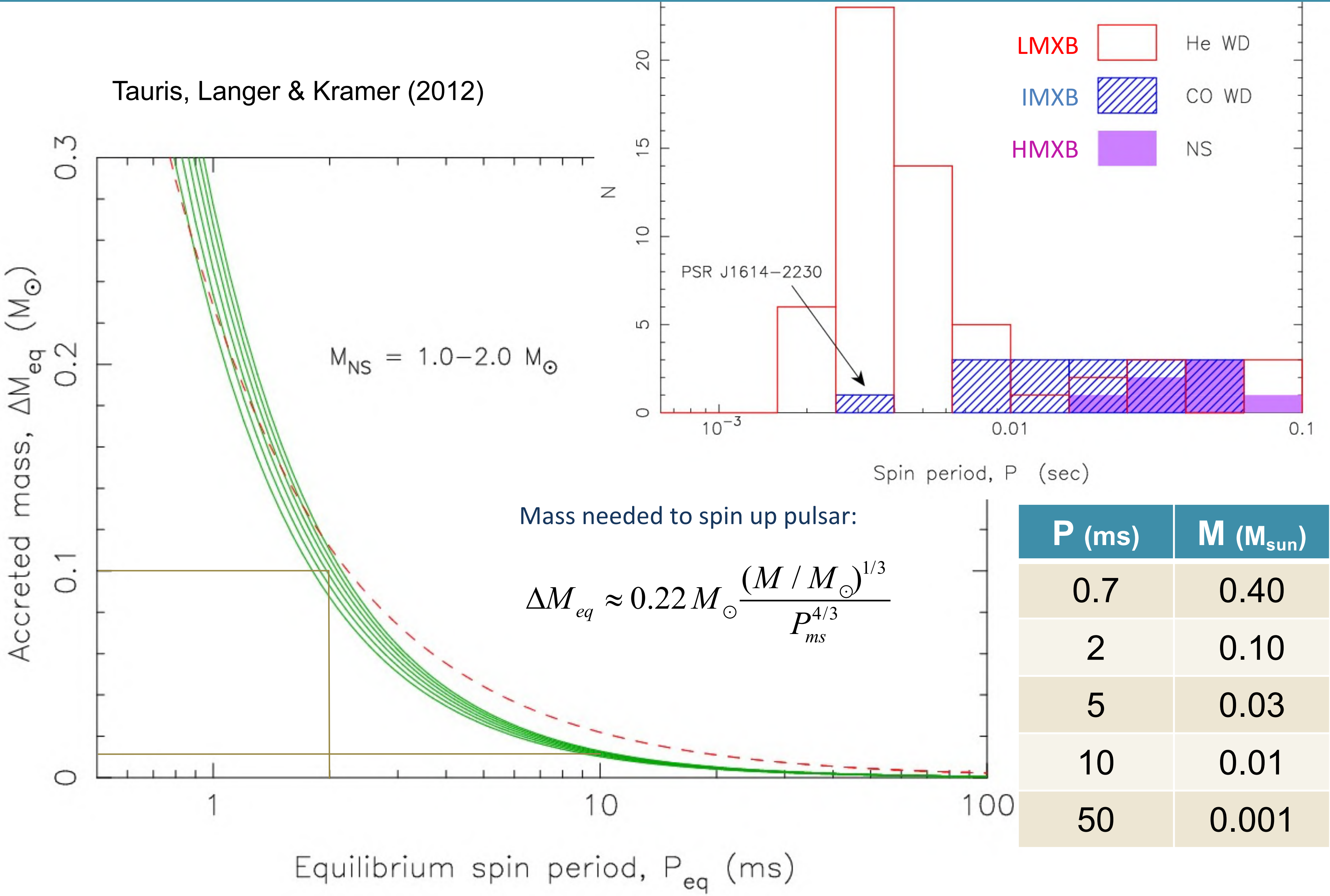
MESA

https://docs.mesastar.org/en



Accreted mass to spin up pulsar

Tauris, Langer & Kramer (2012)

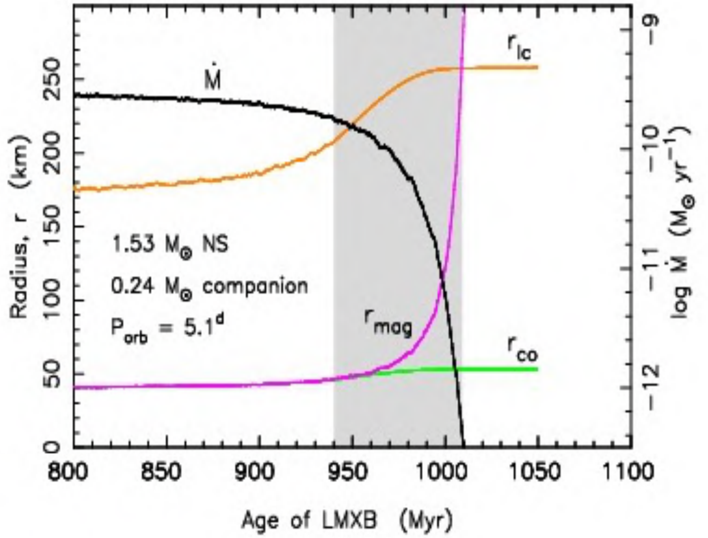


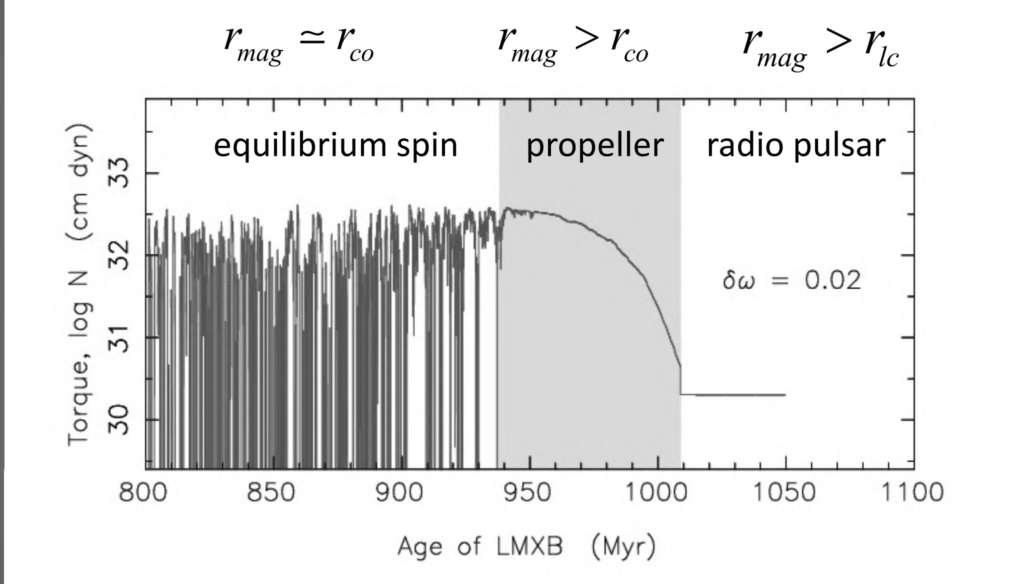
Mass needed to spin up pulsar:

$$\Delta M_{eq} \approx 0.22\,M_\odot \frac{(M/M_\odot)^{1/3}}{P_{ms}^{4/3}}$$

P (ms)	**M** (M_{sun})
0.7	0.40
2	0.10
5	0.03
10	0.01
50	0.001

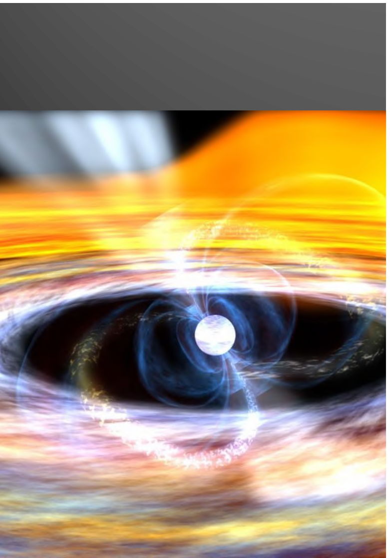
Roche-lobe decoupling phase

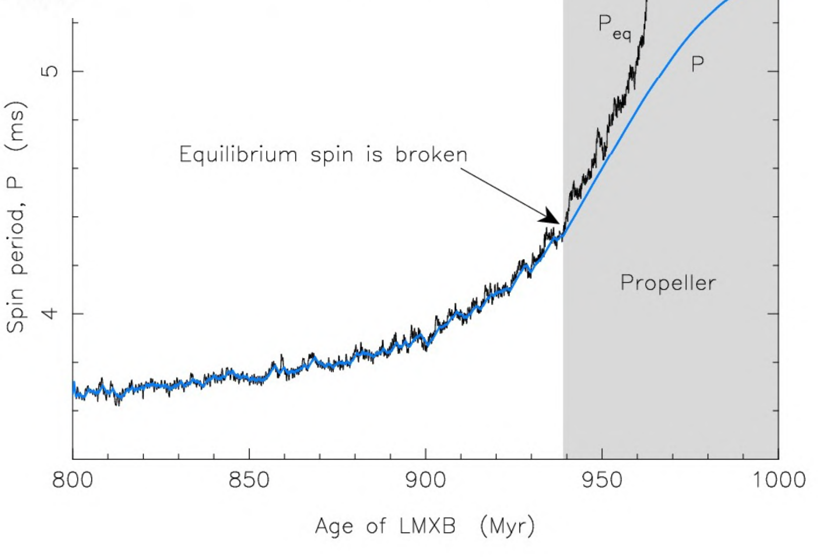


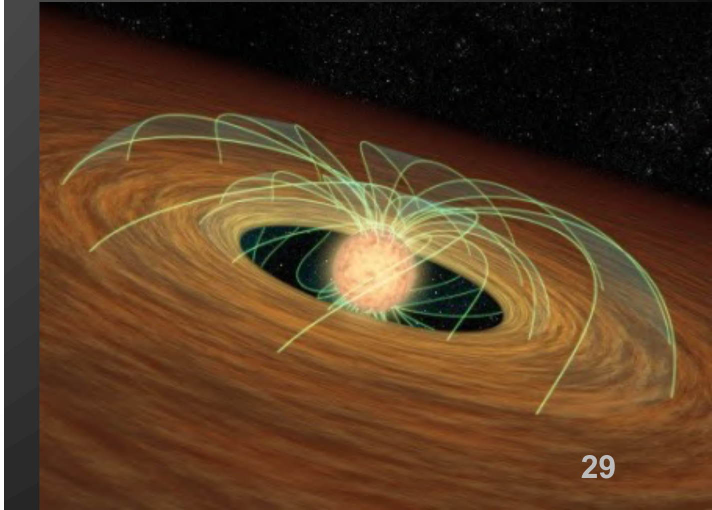


$$N(t) = n(\omega)\left[\dot{M}(t)\sqrt{GM\, r_{mag}(t)}\,\xi + \frac{\mu^2}{9r_{mag}^3(t)}\right] - \frac{\dot{E}_{dipole}(t)}{\Omega(t)}$$

Tauris (2012), Science 335, 561



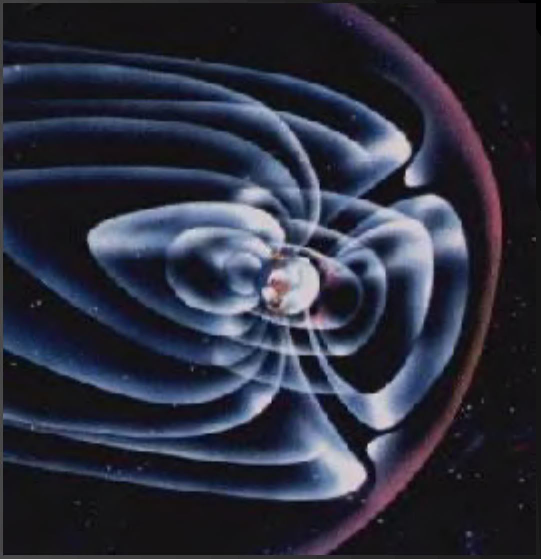




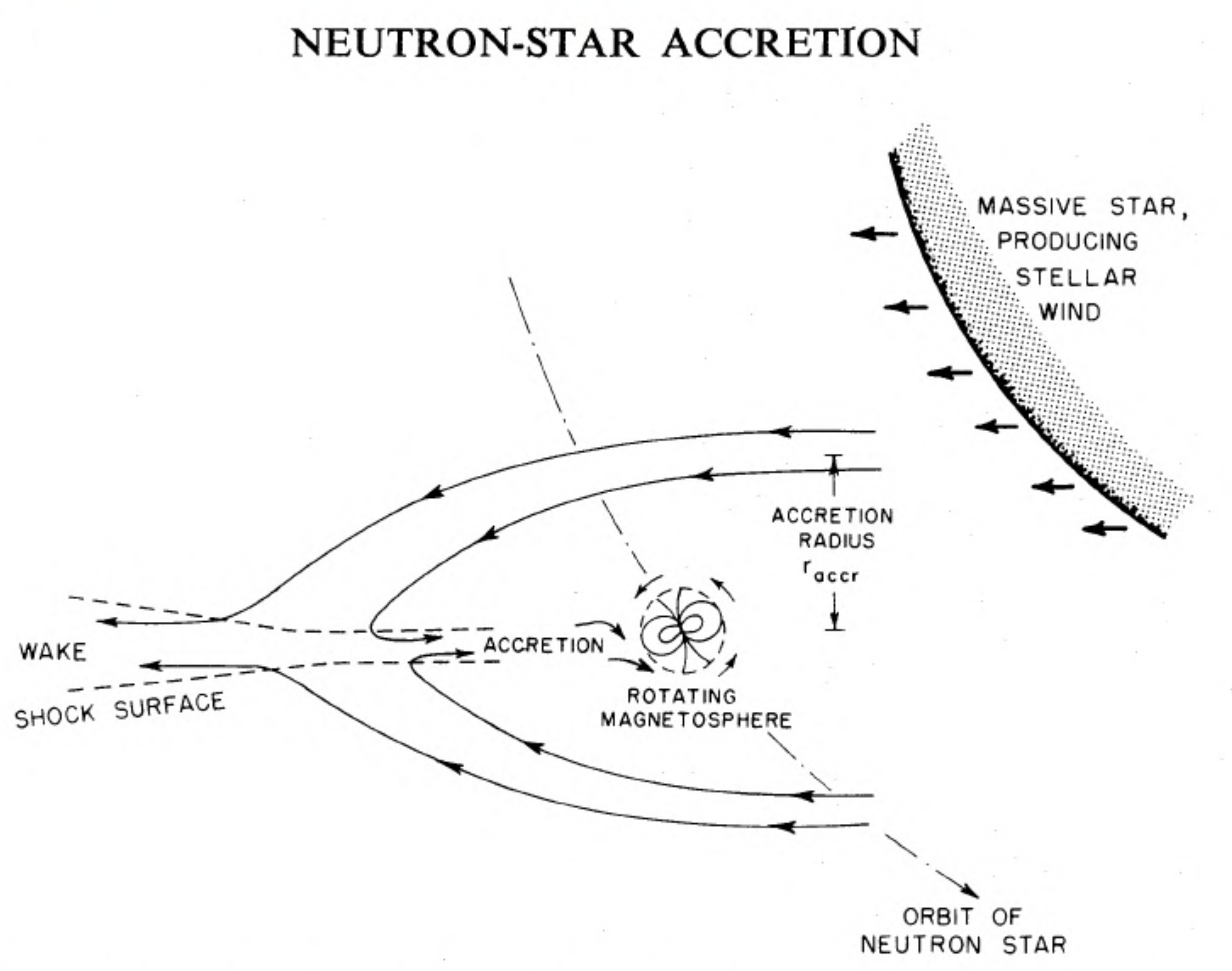
Accretion physics

- four phases of accretion

Self-study



- Introducing the physics of an accreting neutron star
  - Spherical wind accretion
  - Effect of accretion disk

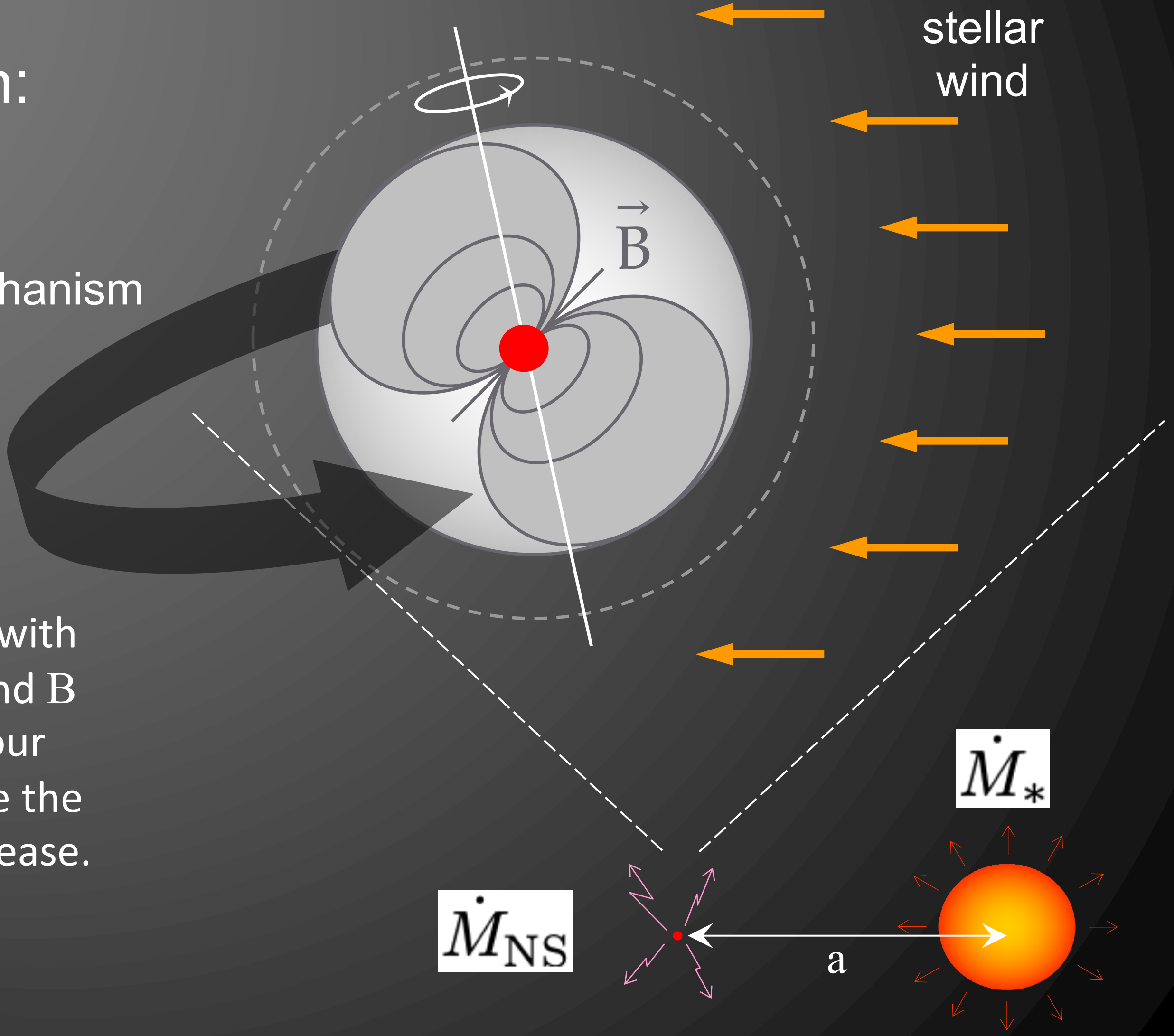


Davidson & Ostriker (1973)

Neutron star accretion

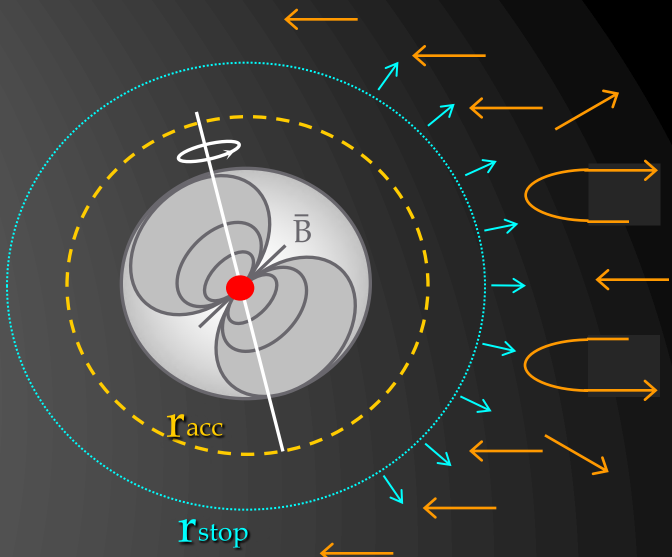
Phases of accretion:

- I. Isolated pulsar
- II. Gunn-Ostriker mechanism
- III. Propeller phase
- IV. Rapid accretion



Consider a young pulsar with initial high values of Ω and B which evolves through four phases of accretion while the values of Ω (and B) decrease.

Phase I



Isolated pulsar: $r_{stop} > r_{acc}$

Wind plasma is stopped by pressure of magneto-dipole radiation outside the radius of gravitational capture. The pulsar evolves as an isolated pulsar.

$$P_{dipole} \approx P_{ram}$$

$$\dot{E}_{dipole} = -\frac{2}{3c^3}|\ddot{m}|^2 \quad \wedge \quad |\ddot{m}| \sim BR^3\Omega^2$$

continuity equation

$$\frac{\dot{E}_{dipole}}{4\pi r_{stop}^2 c} = \frac{2B^2R_{NS}^6\Omega^4/3c^3}{4\pi r_{stop}^2 c} \approx \frac{1}{2}\rho_w v_w^2 = \frac{1}{2}\left(\frac{\dot{M}_*}{4\pi a^2 v_w}\right)v_w^2$$

$$\text{pressure} = \frac{\text{energy}}{\text{volume}}$$

ram pressure of wind

$$\Downarrow$$

$$r_{stop} = \sqrt{\frac{4B^2R_{NS}^6\Omega^4 a^2}{3c^4 v_w \dot{M}_*}}$$

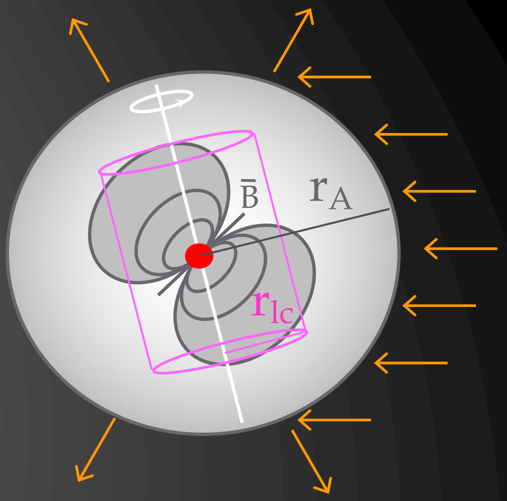
$$r_{acc} \sim \frac{2GM_{NS}}{v_w^2}$$

$$r_{stop} \propto B\,\Omega^2$$

Over time, the pulsar loses rotational energy and Ω decreases, causing r_{stop} to decrease… → Phase II

Phase II

Gunn-Ostriker mechanism: $r_{acc}, r_A > r_{stop}, r_{lc}$



Now $r_{stop} < r_{acc}$. However, the Alfven radius is located outside the light cylinder and matter cannot couple to the magnetosphere with $v > c$. Therefore, matter is accelerated to relativistic energies by magnetodipole waves.

$$\frac{B(r_A)^2}{8\pi} \approx \frac{1}{2}\rho_w\, v_{ff}^2$$

location of Alfven radius:

magnetic energy density = ram pressure

$$B(r_A) = B \cdot \left(\frac{R_{\rm NS}}{r_A}\right)^3$$

perfect dipole

$$v_{ff} = \sqrt{\frac{2GM_{\rm NS}}{r_A}}$$

approx. free-fall radial velocity

Note:

$B(r_A)$ B-field at Alfven radius

$B \equiv B(R_{\rm NS})$ B-field at NS surface

$$\rho_w = \frac{\dot{M}_*}{4\pi a^2\, v_{ff}}$$

continuity equation

$$\dot{M}_{NS} = \frac{\pi r_A^2}{4\pi a^2}\dot{M}_*$$

accretion rate (solid angle accretion)

$$\Rightarrow \boxed{r_A = \left(\frac{1}{32}\frac{B^4 R_{\rm NS}^{12}}{GM_{\rm NS}\dot{M}_{\rm NS}^2}\right)^{1/7}}$$

$$r_A \propto B^{4/7}\,\dot{M}_{\rm NS}^{-2/7}$$

$$r_{lc} \equiv \frac{c}{\Omega}$$

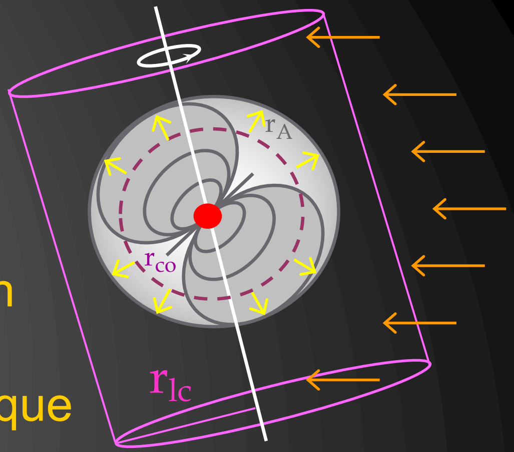
location of light cylinder (distance from spin axis)

$$r_{lc} \propto \Omega^{-1}$$

Over time, Ω decreases, causing r_{lc} to increase… → Phase III

Phase III

Propeller effect: $r_{lc} > r_A > r_{co}$



Accreted matter couples to magnetosphere in super-Keplerian orbits ($F_{centrifugal} > F_{gravitational}$) and thus material piles up near magnetospheric boundary, which creates a strong braking torque (wind carries off ang. mom.)

$$r_{co} = \left(\frac{GM_{NS}}{\Omega^2}\right)^{1/3}$$

co-rotation radius (Keplerian velocity)

$$N = \dot{J}_{spin} \approx \frac{\partial}{\partial t}\left(m r_A^2 \Omega_K\right) = \dot{M}_{NS}\sqrt{GM_{NS}r_A}$$

braking torque

$$\vec{J} = |\vec{r} \times \vec{p}|$$

$$\dot{\Omega} = \frac{\dot{J}_{spin}}{I_{NS}} \quad \wedge \quad \Omega = \frac{2\pi}{P} \quad \Rightarrow \quad \dot{P} \approx \frac{\dot{J}_{spin}P^2}{-2\pi I_{NS}} \propto L_X^{6/7}$$

spin-down rate

$$r_A^{1/2} \propto \dot{M}_{\mathrm{NS}}^{-1/7}$$

$$L_X = \frac{dE_{acc}}{dt} = \frac{GM_{NS}}{R}\dot{M}_{NS} \propto \dot{M}_{NS}$$

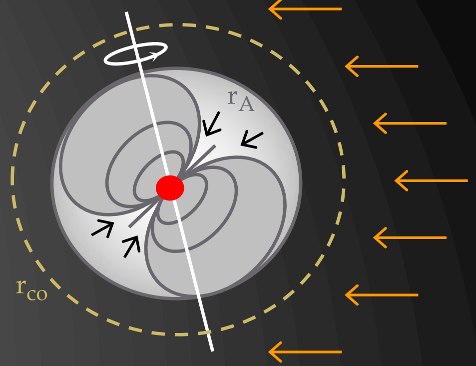
X-ray luminosity

$$r_{co} \propto \Omega^{-2/3}$$

Braking torque causes Ω to decrease… → Phase IV

Phase IV

Neutron star accretion: $r_A < r_{co}$



$$\dot{J}_{spin} = \dot{M}_{NS}\sqrt{GM_{NS}r_A}\;\xi$$

$\xi = 0.01{\sim}0.1,$ if no disk is formed (wind)
$\xi = 1,$ if accretion disk is formed (RLO)

$$\Omega_{NS} = \Omega_K(r = r_A) \quad \Leftrightarrow \quad \frac{2\pi}{P_{eq}} \equiv \sqrt{\frac{GM_{NS}}{r_A^3}}$$

spin equilibrium

equilibrium spin period!

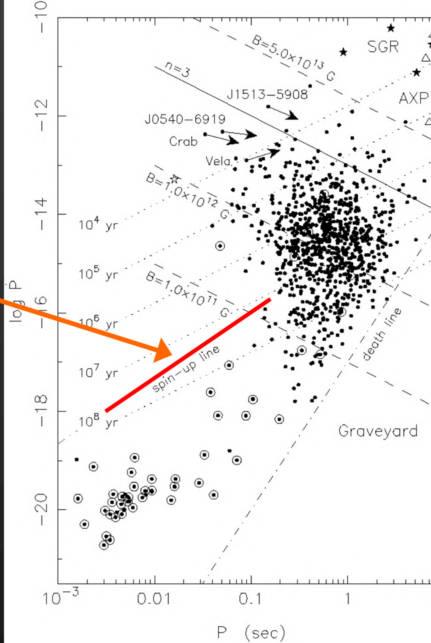
$$\boxed{P_{eq} \propto \dot{M}_{NS}^{-3/7}B^{6/7}R_{NS}^{18/7}M_{NS}^{-5/7}}$$

See eq. for r_A

see spin-up line in (P,$\dot{P}$) diagram

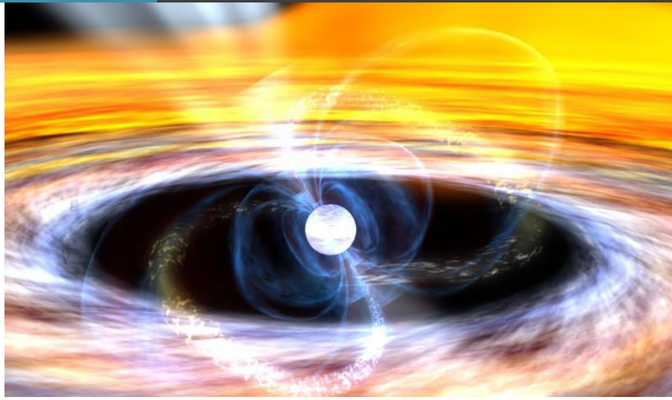
Accretion is possible if $P > P_{eq}$

In equllibrium r_{co} moves alternately inside and outside r_A.



Spin-up line in the PṖ-diagram

Tauris, Langer & Kramer (2012)
MNRAS, 425, 1601



$$P_{eq} = 2\pi\sqrt{\frac{r_{mag}^{3}}{GM}}\,\frac{1}{\omega_c} \quad \wedge \quad r_{mag}(\dot{M},B) \quad \wedge \quad B(P,\dot{P})$$

$$\dot{P} = \frac{2^{1/6}\,G^{5/3}}{\pi^{1/3}c^{3}}\,\frac{\dot{M}M^{5/3}P_{eq}^{4/3}}{I}\cdot(1+\sin^{2}\alpha)\cdot\varphi^{-7/2}\cdot\omega_c^{7/3}$$

spin-up line in $P\dot{P}$ – diagram

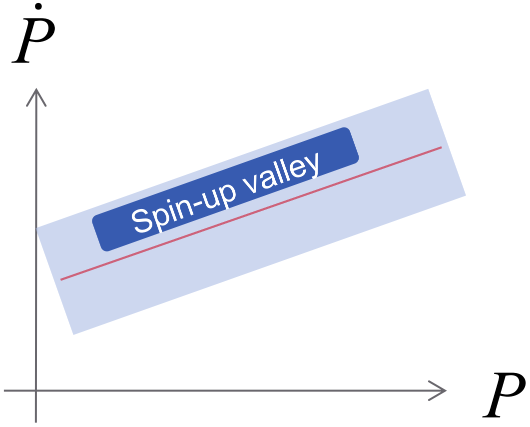
Tauris, Langer & Kramer (2012)

Important!

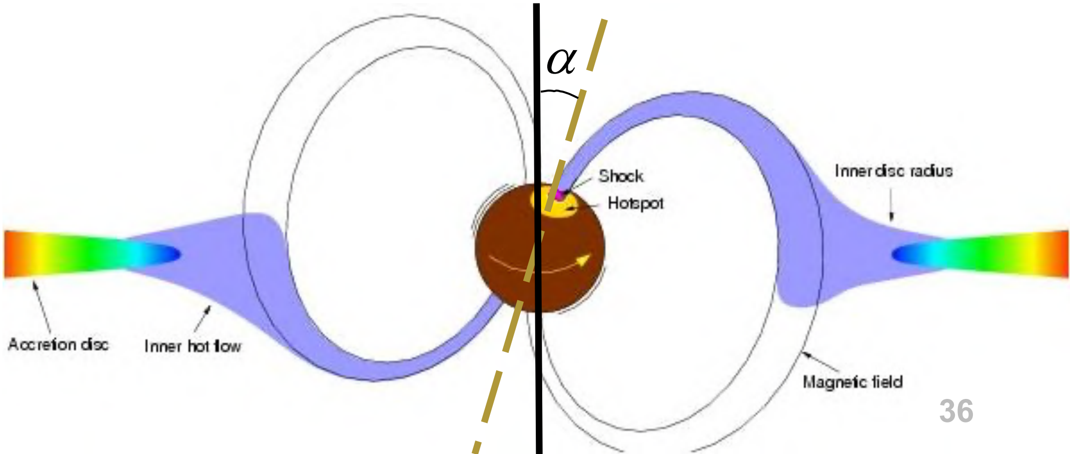
disk –magnetosphere parameters:

$$R_{mag} = \varphi\,R_{Alfven}$$

$$\Omega_{NS} = \omega_c\,\Omega_{mag}^{Kep.}$$

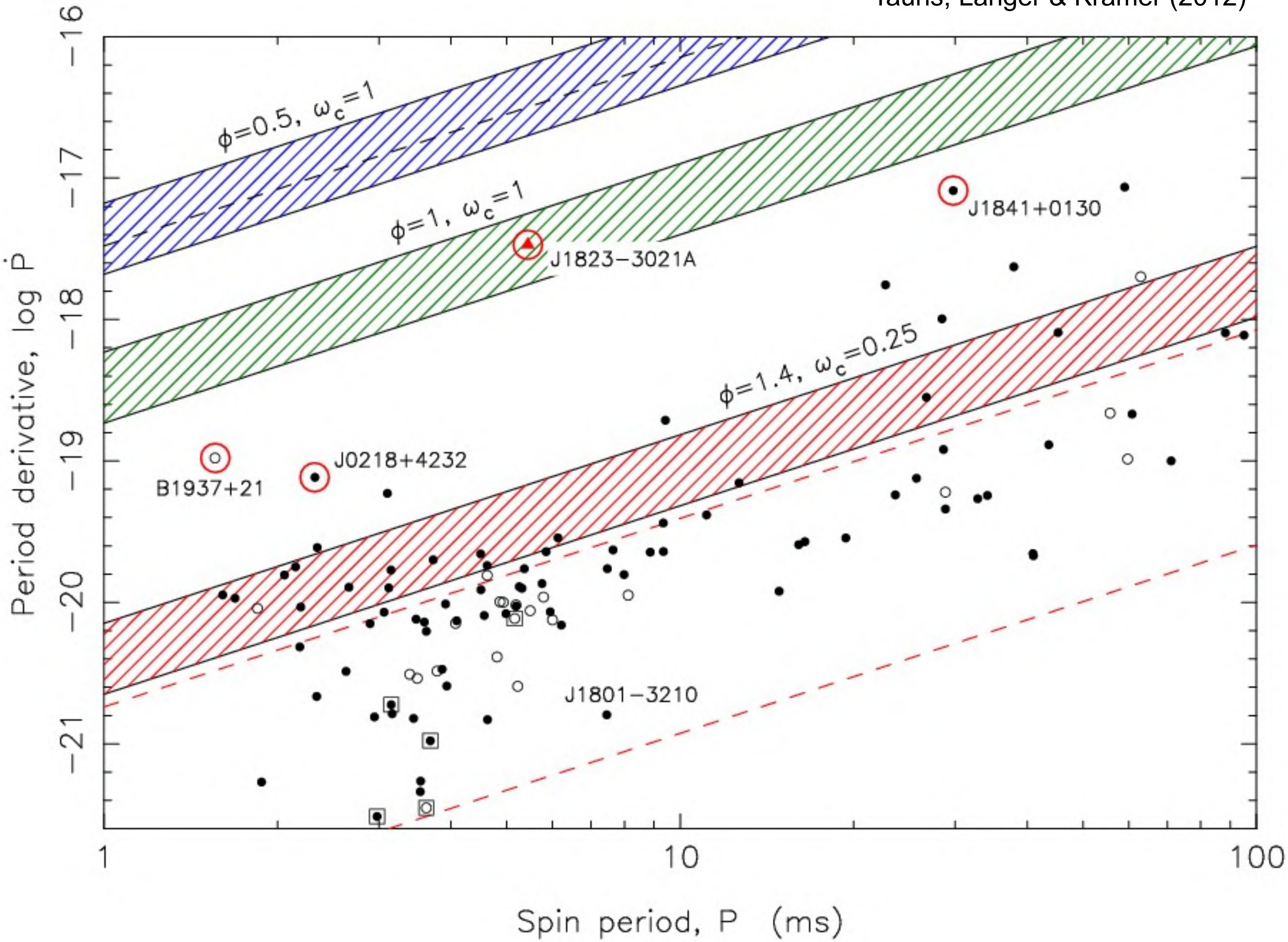


Classical spin-up line
e.g. Bhattacharya & van den Heuvel (1991)



Spin-up line

Tauris, Langer & Kramer (2012)



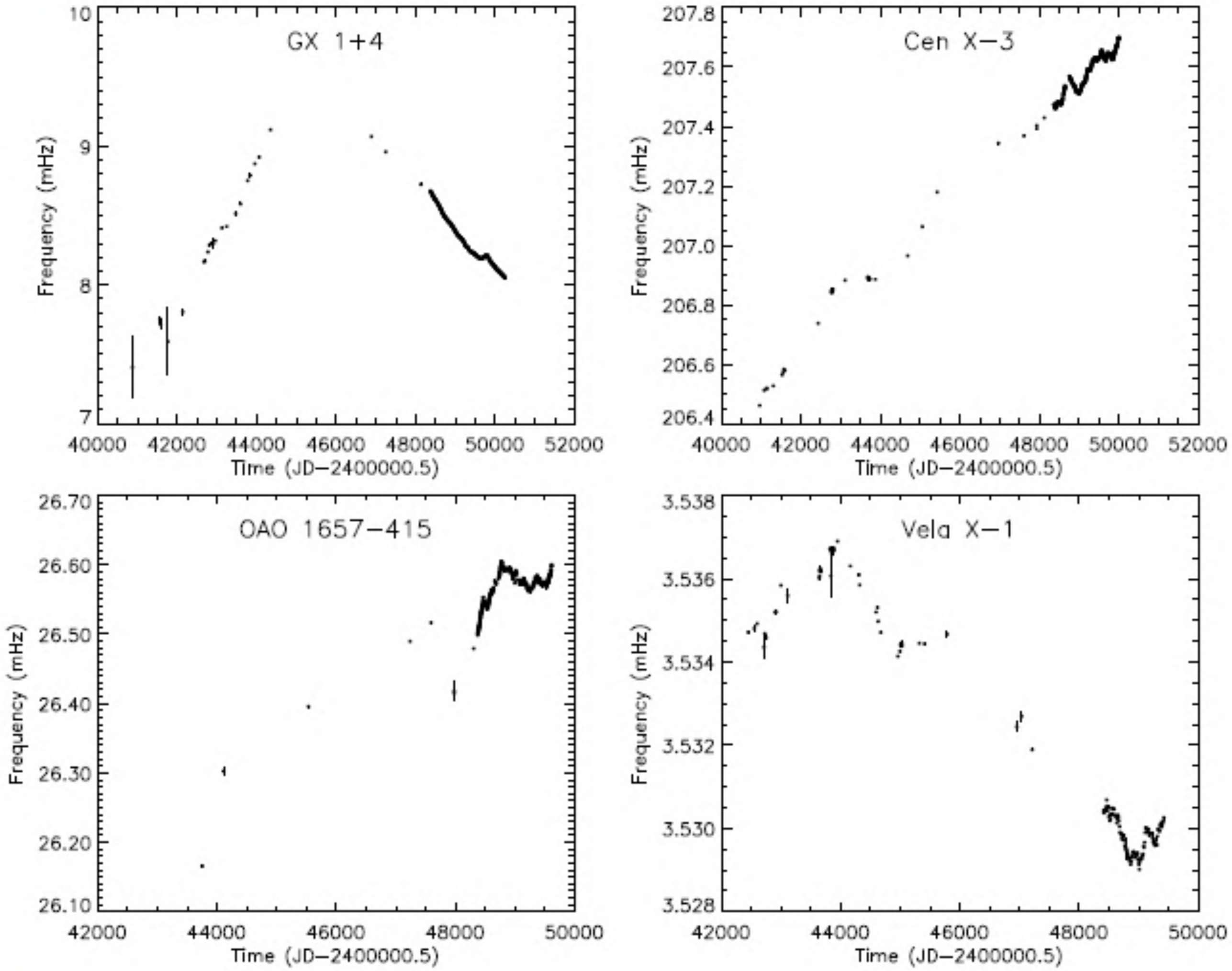
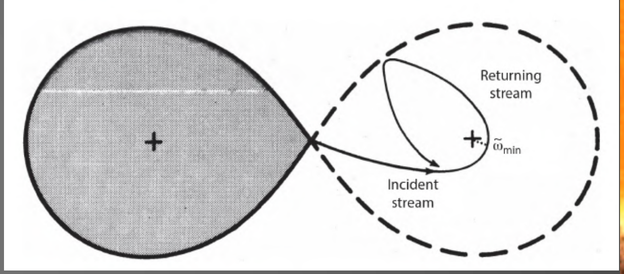
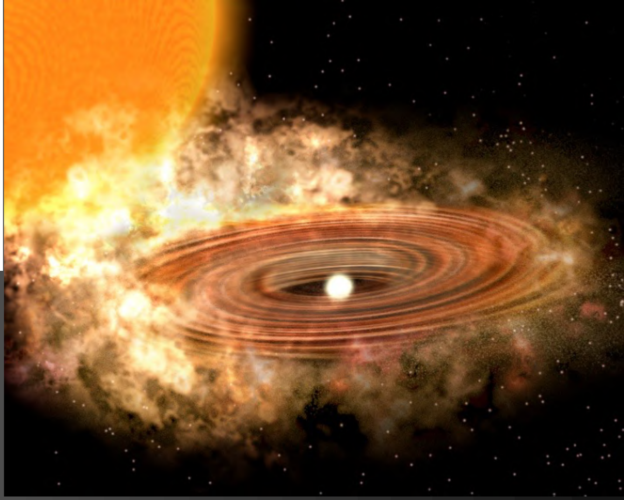


FIG. 6.—Long-term frequency history for all pulsars detected by BATSE that were previously known. The squares show the pre-BATSE data taken from Nagase (1989) and additional references. The line is the BATSE data, which we discuss later in great detail. The long-term frequency history for X-ray pulsars observed by BATSE that were known prior to the *Compton Observatory* launch commences 1991 April. For Her X-1, Cen X-3, Vela X-1, 4U 1538−52, GX 301−2, 4U 0115+634, and EXO 2030+375, all frequencies have been orbitally corrected. For OAO 1657−415, GS 0834−430, 2S 1417−62, and A0535+262, orbital corrections have been applied only to the BATSE observations. No orbital corrections have been applied for 4U 1626−67, GX 1+4, 4U 1145−619, or A1118−615, which have unknown, or incompletely known, orbital elements. The BATSE frequencies for OAO 1657−415, GS 0834−430,

Nagase (1989), Bildsten et al. (1997)

Accretion disks





High specific ang.mom. of accreted gas in binary

→ formation of accretion disk
(ang.mom. is transported outward via viscous stresses)

Turbulent-enhanced viscosity models (e.g. α-model by Shakura & Sunyaev (1973))

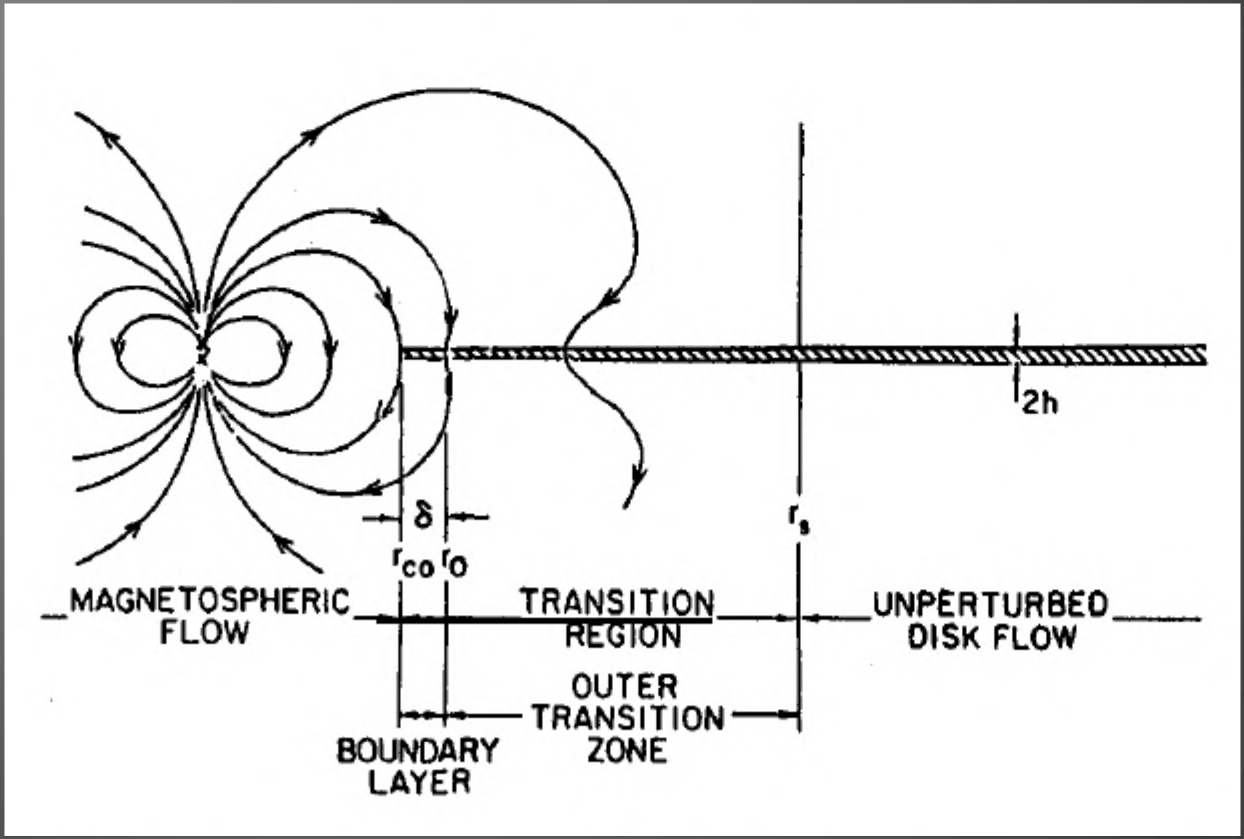
- If accretion rate is < 0.01 $\dot{M}_{Edd}$: thin disk (high opacity) or ADAF (low opacity)
- If accretion rate is about $\dot{M}_{Edd}$: slim disks
- If accretion rate is > $\dot{M}_{Edd}$: torus (with collimated beam of radiation)

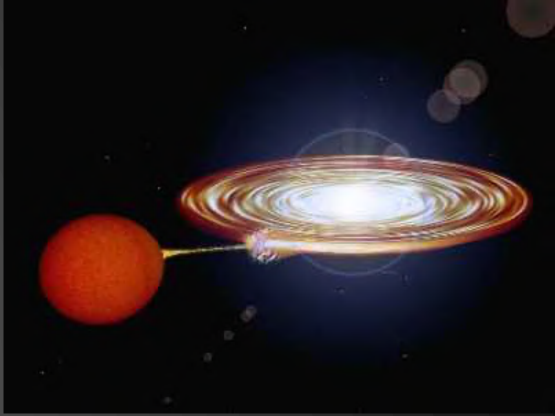
Magnetic stresses truncate the Keplerian disk flow:
- transition zone between disk and magnetosphere

$$R_{magnetosphere} \propto R_{inner\ disk}$$

Spin-up lines in P-P_{dot} diagram depend on nature of accretion disk model (optically thick/thin and gas/radiation pressure dominated)

$$R_{inner\ disk} \propto \dot{M}^{a} \mu^{b} M^{c}$$





Ghosh & Lamb (1979)

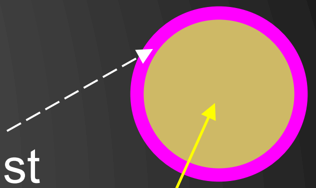
Accretion-induced magnetic field decay

induction equation:

$$\frac{\partial B}{\partial t} = \underbrace{-\frac{c^2}{4\pi}\vec{\nabla}\times\left(\frac{1}{\sigma_{el}}\times\vec{\nabla}\times\vec{B}\right)} + \underbrace{\vec{\nabla}\times(\vec{v}\times\vec{B})}$$

ohmic dissipation (diffusion)

convective transport of accreted material (Hall term)



NS crust

NS core
"$\sigma_{el} = \infty$"

Bad approximation!

$$\frac{\partial B}{\partial t} = -\frac{c^2}{4\pi\sigma_{el}}\nabla^2\vec{B} \quad \Leftrightarrow \quad B = B_0 e^{-t/\tau_D} \quad (\tau_D \approx \mu_0\sigma_{el}L^2)$$

$$\boxed{\sigma_{el} = \sigma_{el}(T,\rho,A,Z,Q)}$$

Note: residual B-field ~10^8 G

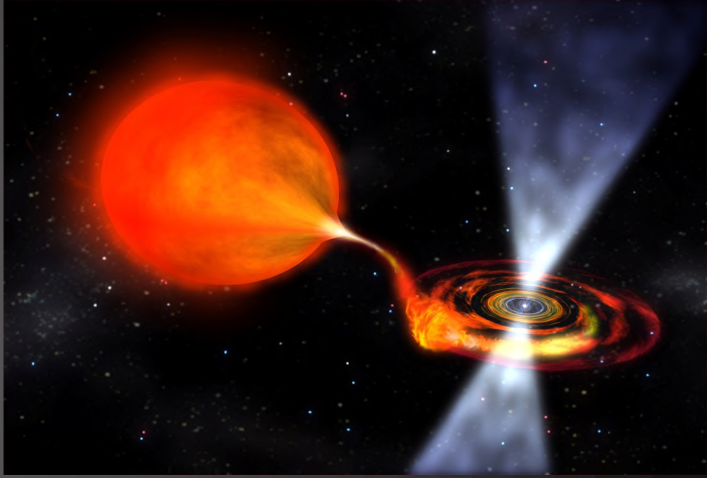
(observed in millisecond pulsars)

due to superconducting interior

Summary

Recyling MSPs

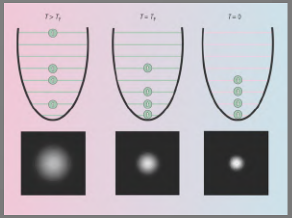
– accretion physics

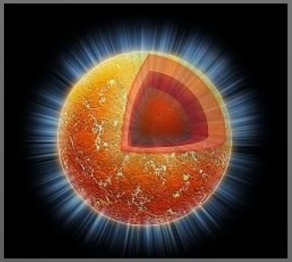


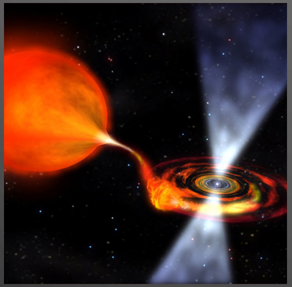
- Detailed LMXB evolution
  - dependence on P_{orb} and M_2
  - relation between M_{WD} and P_{orb} for binary pulsars
  - mass-transfer rate; final neutron star mass
  - equilibrium spin period and spin-up line in P-P_{dot} diagram

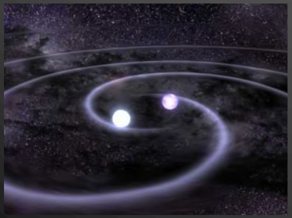
- Accretion physics
  - Four phases of accretion
  - Accretion disks
  - Accretion-induced magnetic field decay

Programme









- ∗ **Introduction**
- ∗ **Degenerate Fermi Gases**

  Non-relativistic and extreme relativistic electron / (n,p,e^-) gases
- ∗ **White Dwarfs**

  Structure, cooling models, observations
- ∗ **Neutron Stars**

  Structure and equation-of-state
- ∗ **Radio Pulsars**

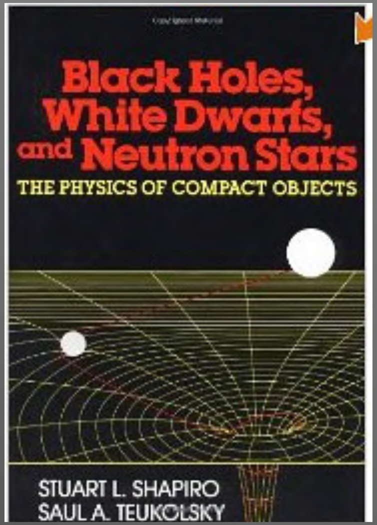
  Characteristics, spin evolution, magnetars, observations, timing
- ∗ **Binary Evolution and Interactions**

  X-ray binaries, accretion, formation of millisecond pulsars, recycling
- ∗ **Black Holes**

  Observations, characteristics and spins
- ∗ **Gravitational Waves**

  Sources and detection, kilonovae
- ∗ **Exam**

Physics of Compact Objects
week 8



Shapiro & Teukolsky (1983), Wiley-Interscience

Curriculum

- Tauris & van den Heuvel (2023), Chapter 7.3 + 14
  (S&T Chapter 18)

- Next lecture: McClintock et al. (2013)
  Tauris & van den Heuvel (2023)
  (Shapiro & Teukolsky Chapter 12 (14))
  (Fabian & Lasenby 2015)
  Aud.5.227

Exercises: # 13, 15, 19

- Monday Oct. 30, 10:15-12:00